Causal Dynamical Triangulations

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Outline

General introduction

- Basic ideas of CDT are derived from the standard QM (QFT)
- Path integral for Quantum Gravity
- Method of DT - 2d example
- Regularization of the theory
- Fractal structure of space-time

Causal Dynamical Triangulations

- Geometry of 1d states and 2d configurations
- CDT - generalization to higher dimensions
- Quantum amplitude - partition function

Numerical results

- Phase structure
- Semiclassical volume distribution

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Postulates

Postulates for Quantum Gravity

- Independence of any a priori fixed background metric. Properties of the observed universe should be **derived** from a theory, and not follow from the assumption.
- Nonperturbative. It should not be a perturbative expansion around some assumed solution of classical equations. Such equations may appear dynamically.
- In the infrared limit it should be described by General Relativity (as a semiclassical limit).

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Approaches

Two approaches:

- String approach: Quantum Gravity is almost a byproduct of a multi-dimensional, supersymmetric “theory of everything”. New objects like strings, (mem)branes. *S*upersymmetry.
- Field theoretical approach: Attempt to quantize gravitational degrees of freedom without introduction of additional variables, extra dimensions or new symmetries. Examples: Dynamical Triangulations (**DT** and **CDT**), Loop Quantum Gravity.

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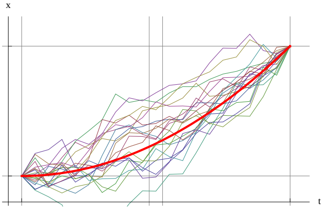
Feynman path integral

Text-book example: non-relativistic point particle in one dimension.



Amplitude of a transition between $|in\rangle$ and $|out\rangle$ states can be represented as a weighted sum over all possible trajectories. On the plot: time is discretized in steps a_t, trajectories are piecewise linear. The red line is the classical trajectory $x_{cl}(t)$.

Path integral cont'd.

To define a path integral it is natural to start with a discretized time in steps a_t and to consider the discretized form of the action. In the continuum limit $a_t \to 0$ the amplitude of a transition between $|in\rangle = |\mathbf{x}_\mathrm{i}\rangle$ and $|out\rangle = |\mathbf{x}_\mathrm{f}\rangle$ states is

$$G(\mathbf{x}_\mathrm{i}, \mathbf{x}_\mathrm{f}, t) := \sum\!\!\!\!\!\!\int_{\text{trajectories: } \mathbf{x}_\mathrm{i} \to \mathbf{x}_\mathrm{f}} \mathrm{e}^{\mathrm{i}S[\mathbf{x}(t)]}$$

where $S[\mathbf{x}(t)]$ is a classical action.
At each (discretized) time step we have a Hilbert space of position states $\{|\mathbf{x}\rangle\}$.

Wick rotation

Wick rotation to imaginary time $t \to \mathrm{i}t_4$ - the weight becomes formally real (positive): $e^{\mathrm{i}S[\mathbf{x}(t)]} \to e^{-S^E[\mathbf{x}(t_4)]}$. In the discretized version we may consider the time spacing $a_t = \alpha a$ and interpret Wick rotation as the analytic continuation $\alpha \to \mathrm{i}\alpha'$ in the complex α plane.
The discretized form of the weight $e^{-S^E[\mathbf{x}(t_4)]}$ requires the action to be dimensionless. In effect the dependence on the dimensionful parameter a can be absorbed by a redefinition of the coupling constants in the action. To reintroduce the dimensionful observables we must consider a critical behaviour of the discretized theory.

Statistical interpretation

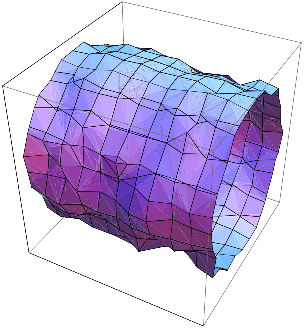
Analogy to Statistical Physics

- Quantum amplitude $\rightarrow$ partition function
- "Classical" trajectory is an average over quantum trajectories in the statistical ensemble of trajectories.
- Discretized theory resembles a statistical theory of a *one-dimensional crystal* with a lattice spacing a.
- Taking the continuum limit $a \rightarrow 0$ requires analyzing the critical behaviour of a theory where the correlation length in the dimensionless units diverges.

Outline

Path integral for Quantum Gravity

Quantum Gravity (without matter) - is a theory of quantum geometry. By analogy we expect the spatial states of the system to be defined as spatial geometries of the universe. We can illustrate the evolution of a one-dimensional closed universe on a plot below:



Joining spatial geometries produces a **space-time geometry**. Trajectory in the path integral is represented as a space-time. In this 2d example the sum over trajectories becomes a (weighted) sum over all two-dimensional surfaces joining the $|in\rangle$ and $|out\rangle$ geometric states, separated by the time T.

Green's function

This is the idealization of the evolution. We would like to find a formulation, where the amplitudes like that for times T_1 and T_2 could be combined to describe the evolution over a time $T_1 + T_2$.

This means that spatial states should form a complete Hilbert space (to be defined). The combined amplitude would then be obtained by integrating (summing) over the space of intermediate states.

Path integral for Quantum Gravity cont'd.

One would hope to make sense of the expression for the amplitude of a transition between the two geometric states (propagator)

$$G(\mathbf{g}_{\mathrm{i}}, \mathbf{g}_{\mathrm{f}}, t) := \sum\!\!\!\!\!\!\int_{\text{geometries: } \mathbf{g}_{\mathrm{i}} \to \mathbf{g}_{\mathrm{f}}} \mathrm{d}\mu[g] \mathrm{e}^{\mathrm{i}S[\mathbf{g}_{\mu\nu}(t')]}$$

So far this expression is formal and requires precisely defining

- the measure $\mathrm{d}\mu[g]$ which should take into account the diffeomorphism invariance
- the domain of integration over space-time geometries (possibly restricting the topology)
- defining the Hilbert space of the spatial geometric states
- introducing a regularization satisfying the points above

Wick rotation

Rotation to imaginary time $t \rightarrow it_4$ - we expect the weight factor to be real:

$$e^{iS[\mathbf{g}(t)]} \rightarrow e^{-S^E[\mathbf{g}(t_4)]}$$

After Wick rotation quantum amplitude becomes a weighted sum over geometric manifolds bounded by the $|in\rangle$ and $|out\rangle$ states.

The simplest form of the action – Hilbert–Einstein action

$$S[\mathbf{g}] = -1/G\,\text{Curvature}(\mathbf{g}) + \lambda\,\text{Volume}(\mathbf{g})$$

where G - gravitational constant, λ - cosmological constant.

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Dynamical Triangulations in 2d

The simplest system to be considered historically was 2d Quantum Gravity. It can be viewed as a toy model for higher-dimensional realizations. Historically it helped to build up a formalism of Dynamical Triangulations (**DT**) and uncovered a number of typical problems.

Assume that (after a Wick rotation) the topology of the space-time is closed. We disregard the boundary terms in the action - Hilbert-Einstein action becomes very simple

$$S[\mathbf{g}] = -\frac{\chi_E}{\kappa} + \lambda V$$

where $\chi_E = 2(1-h)$ is the Euler characteristic of the surface (h - is the number of handles)

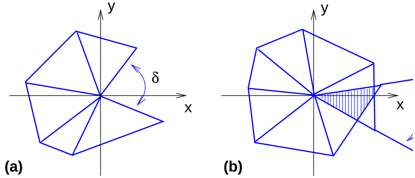
κ is a dimensionless parameter, related to the gravitational constant,

V is a volume of the space-time.

DT in 2d cont'd.

The idea of DT (Dynamical Triangulations) provides a lattice regularization of the space-time geometry. We consider triangulations of space-time by flat equilateral triangles with the edge a (after Wick rotation there is no difference between spatial and time directions).

Curvature is localized in vertices (deficit angle).



Deficit angle δ - (a) positive, (b) - negative. In other points geometry is flat.

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Method of triangulations

Since all triangles are equilateral the Hilbert-Einstein action for a triangulation $\mathcal{T}$ simplifies

$$S_{HE}(\mathcal{T}) = -\frac{\chi_E(\mathcal{T})}{\kappa} + \Lambda N(\mathcal{T})$$

Here $N(\mathcal{T})$ is a number of triangles in a triangulation and Λ is the bare cosmological constant.
The regularized space-time trajectory becomes a closed simplicial manifold characterized by invariants. We assume all edges to be equal, but a number of triangles $o(v)$ meeting at a common vertex v may change.
This form is coordinate independent. No over-counting, since two different manifolds represent two different geometries. Sum over trajectories becomes a sum over simplicial manifolds.

Topological restrictions - entropy

The problem of calculating the quantum amplitude of gravity is reduced to a combinatorial problem of counting the number of inequivalent realizations of the 2d space-time geometry characterized by two topological invariants: Euler characteristic χ_E and volume N.

Simple argument: if we do not restrict the topology of space-time manifolds the number of inequivalent geometries behaves as $N!$ (factorially). As a consequence the expression

$$\mathcal{Z} = \sum_{\mathcal{T}} e^{-S_{HE}(\mathcal{T})}$$

for the amplitude is not even Borel summable.

We restrict the class of admissible space-time topologies $\mathcal{T}$ to those with the simplest (spherical) topology.

Topological structure of the spatial Universe

The reduction of the space-time topology is related to the problem : what is the topology of the spatial Universe.
Question: Is the spatial Universe connected (closed) or can it split into disconnected fragments? Topological splitting (handles) means that our naive ideas about the topology of space-time are wrong and that perhaps we should invent a mechanism controlling such process.
If the amplitude is not Borel summable a theory may have (infinitely) many non-perturbative versions.
As will be shown below even restricting the space-time topology may not be enough to obtain a simple theory.

Entropy - cont'd.

Let us consider triangulations with $\chi_E = 0$. The expression for a 2d quantum amplitude becomes.

$$\mathcal{Z} = \sum_N \mathrm{e}^{-\Lambda N} \mathcal{N}(N)$$

where $\mathcal{N}(N)$ is a number of inequivalent triangulations built of N triangles. This number (entropy) behaves as

$$\mathcal{N}(N) = \mathrm{e}^{\Lambda_c N} N^{\gamma - 3}(1 + O(1/N))$$

where Λ_c is a (non-universal) bare critical cosmological constant and γ - a universal critical exponent.
Universality means that the same typical behaviour will appear in more complicated discretizations using polygons rather than triangles. Theory is defined for $\Lambda > \Lambda_c$ and $\Lambda_{eff} = \Lambda - \Lambda_c$ sets the scale for all physical observables which can be defined in a theory.

Continuum limit, spatial states

Continuum limit corresponds to $\Lambda_{eff} \to 0$, where the amplitude is dominated by large N. In this limit we may reintroduce the dimensionful parameters

$$\Lambda_{eff} = \lambda a^2, \quad a^2 N \propto V$$

The simplest amplitude which can be studied is a manifold with a topology of a planar disc, with a closed one-dimensional boundary of (integer) length k bounding the triangulated surface built of N triangles.
The amplitude in this case has a well defined continuum limit for $\Lambda_{eff} \to 0$ (and $V = Na^2$ fixed) with a boundary length $L = ka$, where L is a physical length of the boundary. Continuum limit corresponds to a two-dimensional conformal field theory.

Spatial states, time evolution

After a Wick rotation the identification of space and time directions was lost. We may try to reintroduce these quantities by interpreting the boundary of a disc as a possible initial spatial state. The initial state would be characterized by the geometry of a closed circle (polygon) with a length k and a topology of S^1. We assume that a boundary of the disc defines the state of the Universe at a time $t = 0$.

We can define a distance between a point of the triangulation and a closed loop on a triangulation as a minimal number of steps along the edges between this point and any point of a loop. This definition can be extended to define a distance between loops.

Time evolution cont'd.

The Hilbert space of states is represented by closed loops with an integer length $k > 0$. To define the elementary evolution step in time we consider all possible ways to connect two loops in such a way that they are separated by a unit distance $\Delta t = 1$. If we succeed we iterate the procedure.

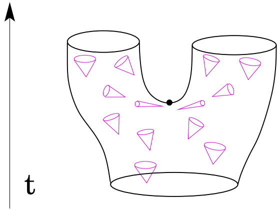
The amplitude for a single step $\langle k_1|M_1|k_2\rangle$ is obtained by summing over all manifolds with two boundaries, at a fixed distance $\Delta t = 1$ and weighted by a factor $\exp(-\Lambda N)$. Considering all manifolds contributing to the amplitude one realizes that the naive idea that geometric evolution is smooth is not true.

Outline

Fractal structure in EDT

Causality problem - baby universes

Looking at structures separated by one time step from the initial loop we see that a generic situation corresponds to a creation of baby universes.



This is independent of how precisely we define the “time flow” in DT. It means also that the concept of a single universe with a well defined circular topology is unstable against breaking into many universes, even though the space-time topology is spherical.

We may keep only one universe and sum over the remaining structures but this means that in the process of evolution we loose information about other baby universes.

Disregarding baby universes

It is possible to calculate the amplitude in such a way that we always keep one incoming loop and one outgoing loop (integrating out other baby universes). The price one pays is that on a large time scale time T scales in the anomalous (non-canonical) way as $\sqrt[4]{\Lambda_{eff}}\,T \approx \sqrt{a}T \propto T/N^{1/4}$.

Conclusion We must reduce topological structures included in a summation over space-times if we want to keep the canonical relations between space and time in the continuum limit.

Fractal dimensions

On random geometric structures we should define what we mean by the (averaged) dimension of a manifold. Two possible definitions:

- Hausdorff dimension d_H. For a ball with a radius $r >> 1$ we measure the number of points inside the ball:

$$\langle N(r) \rangle \sim r^{d_H}$$

For a ball with a finite volume V we should have

$$\langle r \rangle_V \propto V^{1/d_H}$$

- Spectral dimension d_S. We define a diffusion process on a manifold in a pseudo-time σ. Return probability characterizes the heat kernel of the Laplacian

$$\langle P(\sigma) \rangle \propto 1/\sigma^{d_S/2}$$

For a 2d space-time discussed above $d_H = 4$, $d_S = 2$.

Conclusions

- A sum over topologies (sum over h) is badly divergent, since for large N the number of possible manifolds grows factorially. In practice we have to restrict a class of topologies to $h = 1$ (planar surfaces).
- For surfaces with a fixed genus h the number of possible manifolds grows exponentially with N

$$\mathcal{N}(N) \propto e^{\Lambda_c N} N^{\gamma_h - 3}(1 + \cdots)$$

  Theory is defined only for $\Lambda > \Lambda_c$, where

$$\langle N \rangle \propto 1/(\Lambda - \Lambda_c)$$

  .
- Spatial states with S^1 topology are unstable against a formation of baby-universes.

Outline

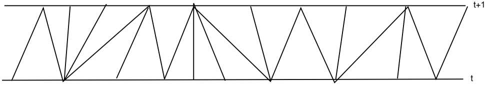
Causality

A solution to the problems discussed before : eliminate causal singularities by explicitly suppressing a possibility of the baby universe formation.

We consider only space times which admit a global time foliation and for which a topology of the spatial geometry at a fixed time is constant (say S^1 in 2d).

We use the general idea of the *Dynamical Triangulations* - space-time structures obtained by gluing together simplices - **Causal Dynamical Triangulations**.

Loops in the neighboring time layers are connected by triangles pointing up or down.



CDT in 1+1 dimensions

- States: Closed loops with the (integer) length $k > 0$. k is a spatial volume of the Universe.
- Connecting loop states: Each link of the loop is a base of a triangle pointing up in time (a $\{2,1\}$ triangle) and a triangle pointing down (a $\{1,2\}$ triangle). Triangles are glued along the time-like edges in all possible ways.
- Elementary single step amplitude: A sum over all ways to connect two loops with lengths n and m using the two types of triangles.We need n $\{2,1\}$ triangles and m $\{1,2\}$ triangles. Even in 2d the number of possible ways to connect two states is exponentially large ($\propto 2^{n+m}$). We need a factor $\mathrm{e}^{-\Lambda(n+m)}$ with $\Lambda > \log 2$ to suppress the entropy.

CDT

- Coupling constants: In 2d the only relevant coupling constant is the cosmological constant $\Lambda_{eff} = \Lambda - \Lambda_c \propto a^2$. Gravitational constant is eliminated because of a topological identity (Euler characteristic).
- The 2d model can be analytically solved and we can study the large N and large T limit (continuum limit $\Lambda_{eff} \to 0$).
- In the continuum limit both the spatial volume of Universe ($L = ka$) and the time ($t = Ta$) scale canonically (i.e. $\propto a$).
- The continuum amplitude reproduces the 2d QG amplitude in a proper time gauge.
- For a two-dimensional system $d_H = 2$ and $d_S = 2$.

Outline

CDT in d+1 dimensions

The model of Causal Dynamical triangulations can be easily generalized to higher dimensions. The difference is that now we should describe the evolution in time of a d-dimensional spatial geometry. As before, in the path integral, we consider only geometries admitting a global time foliation for which the spatial topology is preserved in time. We assume that the spatial topology of the Universe is spherical (S^d) . The model is regularized using the method of Dynamical Triangulations. We assume that we may perform a Wick rotation on each space-time configuration.

Hilbert space of geometric states

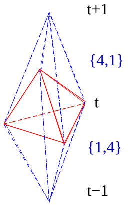
The Hilbert space of spatial geometric states is now much reacher. It consists of all inequivalent d-dimensional geometries with a spherical topology. To count these states in a diffeomorphism-invariant way we use the idea of a triangulation: each state is represented as a d-dimensional simplicial manifold constructed from regular d-simplices with the edge length a_s. Different triangulations correspond to different (orthogonal) states.

Each state is the eigenstate of the spatial volume operator (counting the number n of d-simplices). The Hilbert space of states breaks in a natural way into subspaces with different n. The number of states in each subspace grows exponentially with n.

Path integral

We define the path integral by considering all possible ways to connect a triangulation $\mathcal{T}$ at time t with a triangulation $\mathcal{T}'$ at $t+1$.
Consider the most interesting case $d=3$, where the spatial geometry is represented by a triangulated spherical manifold built from regular tetrahedra. These geometric states must be connected in the neighbouring times.

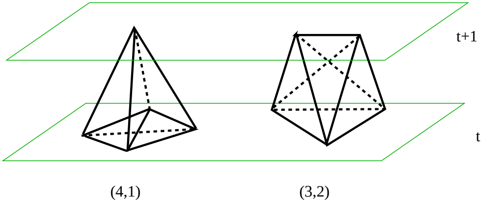
Each tetrahedron becomes a base of a pair of $\{4,1\}$ and $\{1,4\}$ simplices, pointing up or down in t. The lengths of edges in time direction are a_t (may be different than a_s).



Manifold construction in 4d CDT

We need two more types of simplices: $\{3,2\}$ and $\{2,3\}$.

Simplices $\{3,2\}$ and $\{2,3\}$ form a "layer" gluing together states at t and $t+1$.



Path integral

The construction presented above is used to build space-time trajectories. A trajectory is a sequence of 3-dimensional geometric states at integer times t, each represented as a manifold built of $n(t)$ regular tetrahedra, extended to a 4-dimensional simplicial manifold using the construction presented before. Notice that a volume $n(t)$ at a spatial slice t is proportional to the number $N_4^{\{4,1\}}$ of simplices $\{4,1\}$ (or equivalently the number $N_4^{\{1,4\}}$ of simplices $\{1,4\}$). A large part of the space-time volume is contained in $\{3,2\}$ and $\{2,3\}$ 4-simplices, necessary to close the geometry.
The spatial states at integer times are separated by the layer of $\{3,2\}$ and $\{2,3\}$ simplices with a topology S^3. This layer is responsible for a transfer of information between the neighboring three-dimensional states.

Manifold construction in 4d CDT cont'd.

Each four-dimensional manfold can be characterized by a set of "global" numbers

- $N_4^{\{4,1\}}$ - number of $\{4,1\}$ and $\{1,4\}$ simplices.
- $N_4^{\{3,2\}}$ - number of $\{3,2\}$ and $\{2,3\}$ simplices.
- N_0 - number of vertices (0-simplices).
- T - time period.

All other "global" numbers can be expressed as linear combinations of these numbers because of topological identities.
Details about the space-time geometry are contained in a local information defining the way manifold is glued together.

Curvature

The trajectories will be weighted by a factor $\mathrm{e}^{-S_{HE}}$, where S_{HE} is the discretized Hilbert-Einstein action

$$S[\mathbf{g}] = -1/G\,\text{Curvature}(\mathbf{g}) + \lambda\,\text{Volume}(\mathbf{g})$$

The curvature contribution can be expressed by a sum of deficit angles around the $d-2$-dimensional hinges (triangles).
The simplicity of the geometric construction of CDT means that the action can be expressed as a linear combination of three "global" numbers multiplied by three bare coupling constants.

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Partition function

Hilbert-Einstein action is parametrized by 3 bare parameters: $\kappa_0,\ \kappa_4$ and Δ.

$$S_{HE} = -(\kappa_0 + 6\Delta)N_0 + \kappa_4(N_4^{\{4,1\}} + N_4^{\{3,2\}}) + \Delta(2N_4^{\{4,1\}} + N_4^{\{3,2\}})$$

The model cannot be solved exactly. We are forced to study the properties of the model using numerical methods (Monte Carlo simulations). This means that we have to use from the start a Wick-rotated version of the model with imaginary time. We interpret the real weight factor $\mathrm{e}^{-S_{HE}(\mathcal{T})}$ as a probability to observe a triangulation $\mathcal{T}$.

Parameters of the H-E action

Physical properties of the system are determined by values of bare coupling constants

- $\kappa_4 - \kappa_4^{crit}(\kappa_0, \Delta)$ - related to the average $\langle N_4 \rangle$.
- κ_0 - related to the bare gravitational constant.
- Δ - related to asymmetry between a_s and a_t.

In our approach we study properties of systems with finite volumes $N_4 \to \infty$, which is equivalent to $\kappa_4 \to \kappa_4^{crit}$.
This limit does not need to give a physically acceptable continuum limit (case of Euclidean DT). The number of independent triangulations as a function of $N_4 = N_4^{\{4,1\}} + N_4^{\{3,2\}}$ typically behaves as $\mathrm{e}^{\kappa_4^{crit} N_4}$.

In the three-dimensional parameter space $\kappa_4 = \kappa_4^{crit}(\kappa_0, \Delta)$ describes a critical surface. Approaching this surface (from above) corresponds to a large-volume limit where $N_4 = N_4^{\{4,1\}} + N_4^{\{3,2\}}$ becomes large (infinite).

- Instead of analyzing the model in the critical regime ($\kappa_4 \to \kappa_4^{crit}$) we consider a sequence of fixed volume amplitudes with $N_4 \to \infty$ using the property

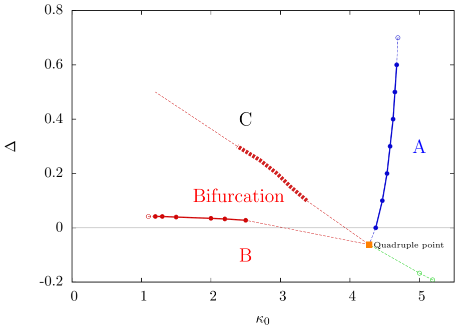
$$\kappa_4 - \kappa_4^{crit} \propto \frac{1}{N_4}$$

In the infinite volume limit we effectively reduce the space of bare parameters to a set $\{\kappa_0, \Delta\}$.

Outline

Approximate phase diagram of CDT

The CDT model in 4d is analyzed using numerical Monte Carlo methods. We find a surprisingly reach phase structure.



Phase structure for $\kappa_4 >\approx \kappa_4^{crit}(\kappa_0, \Delta)$ (infinite volume limit).

Red lines - phase transitions. Perhaps a quadruple point.

Observables

- Numerical simulations produce space-time configurations. They are like quantum trajectories of a particle, in general we do not expect them to give a direct information about a classical space-time.
- Our formulation is background independent: we do not have any reference frame.
- It is not obvious how to define the relevant observables telling us something about a semi-classical limit.
- As will be explained in other talks the observables we measure are not very close to the intuition about the classical geometry.

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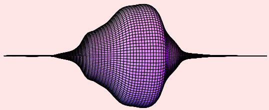
Analyzing the average geometry

The basic object measured in our numerical experiments is the distribution of the spatial volume $n(t) = N^{\{4,1\}}(t)$. This is similar to the scale factor used in many approaches to gravity. Notice however that in our case each value of $n(t)$ represents a whole space of possible geometric states with a volume $n(t)$. The concept of a state $|n(t)\rangle$ may be misleading, one should rather think about a projection $|n(t)\rangle\langle n(t)|$
The behavior of a typical volume distributions is very different in different phases. The most interesting physically is the phase C (de Sitter phase).

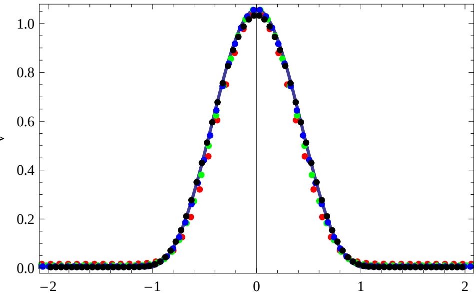
Snapshot of a typical configuration in C phase



One typical configuration. Distribution of a spatial volume $n(t)$ as a function of (imaginary) time t looks like a quantum fluctuation over a semi-classical background. Configuration consists of a “stalk” of the cut-off size and a “blob”. Center of the blob can shift. We fix the “center of mass” to be at zero time.

Scaling in the C phase

Example of the scaling analysis in the C phase. We average the volume distributions over independent configurations and extract the limiting semi-classical distribution, where volume is scaled by by a factor $N_4^{3/4}$ an the time by a factor $N_4^{1/4}$. This behaviour corresponds to the Hausdorff dimension $d_H = 4$.



The continuous line corresponds to $f(\tau) \propto \cos^3(\tau)$ (4d sphere).

Other phases

Volume profile is different in the other phases:

- phase A is characterized by the lack of correlation between volumes of the neighboring slices. Configurations look like a random distribution of volume.
- Phase B is a phase where the blob collapses to a single slice: it can be viewed as a spontaneous compactification of the time dependence.
- In a newly discovered bifurcation phase we observe a blob, but the scaling is different than in the phase C.

There are many other differences between the phases.

Effective model

The semi-classical distribution of volume can be obtained in the effective model, where we include only volume as a degree of freedom. We can measure the effective action of the model. Details will be given in other talks.
The effective model can be used to determine physical parameters of the CDT model as a function of bare parameters. They are different in different phases. The effective action in the phase C looks like a discretization of the mini-superspace model.
The most important (and the most difficult) problem is to analyze the behavior of the model near phase transitions. This is the subject of the present studies. Notice that in a 4d gravity the gravitational coupling constant is dimensionfull. Finding a correct continuum limit is therefore a delicate problem.

Phase transitions

- The phase transition, which is interesting for us must be second or higher order. At such a transition the dimensionless correlation length of some physical observable(s) diverges. We would like the observable to be the curvature.
- A phase transition between A and C is first order, so it is not a good candidate to represent continuum gravity.

- The possible place to find a new theory is along a transition between C and a bifurcation phase. This is the phase with properties we still do not understand completely. We suspect that at the phase transition we observe a spontaneous change of the effective metric from Lorentzian to Euclidean. The problem is currently analyzed.
- The biggest challenge is to study the behavior of the model near a quadruple point, which may be a candidate for the non-perturbative fixed point of Quantum Gravity.

Thank you