Jakub Gizbert-Studnicki

in collaboration with

Jan Ambjørn, Daniel Coumbe, Andrzej Görlich and Jerzy Jurkiewicz

A Spontaneous Signature Change in CDT Quantum Gravity?

Quantum Gravity in Cracow^4

May 2015

UNIWERSYTET JAGIELLOŃSKI

NATIONAL SCIENCE CENTRE

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Outline

- ✧ *CDT*
- ✧ *Effective action*
- ✧ *Effective action in Phase C*
- ✧ *Transfer matrix method*
- ✧ *Effective action in Phases A & B*
- ✧ *Phase transitions*
- ✧ *Bifurcation phase*
- ✧ *Signature change*

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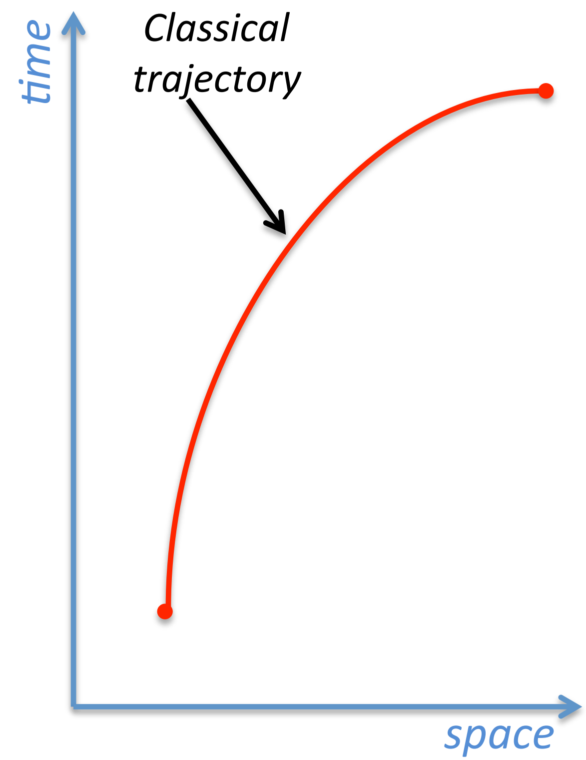
- ✧ *Quantum mechanics: all trajectories (paths) contribute to transition amplitude (weight/phase factor depends on the action)*
- ✧ *Path integral is defined by a discretization of time (regularization)*



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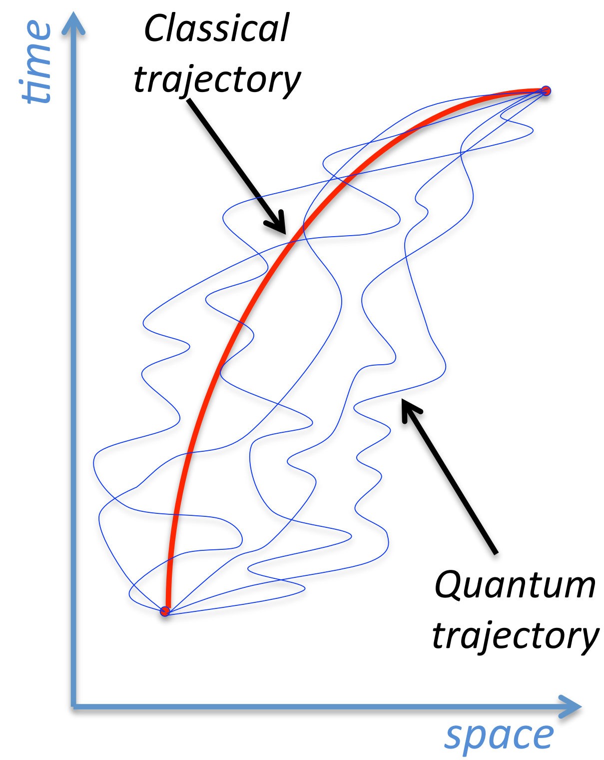

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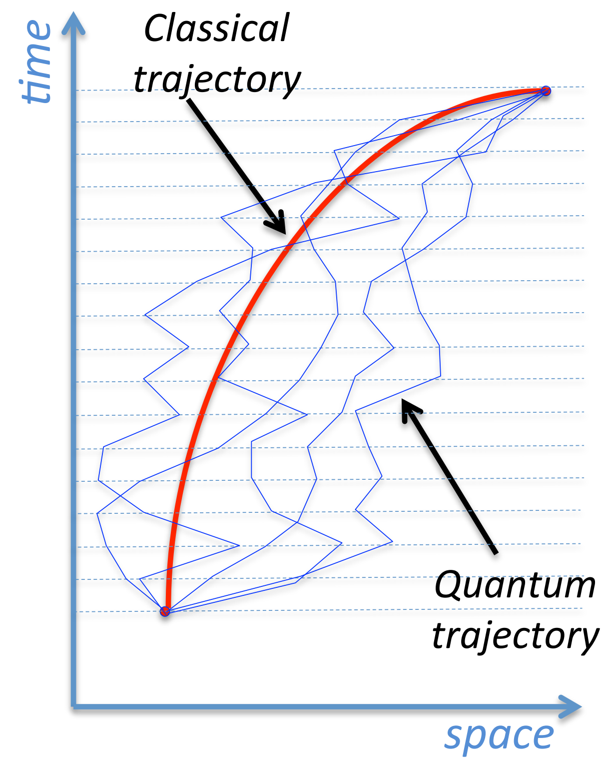

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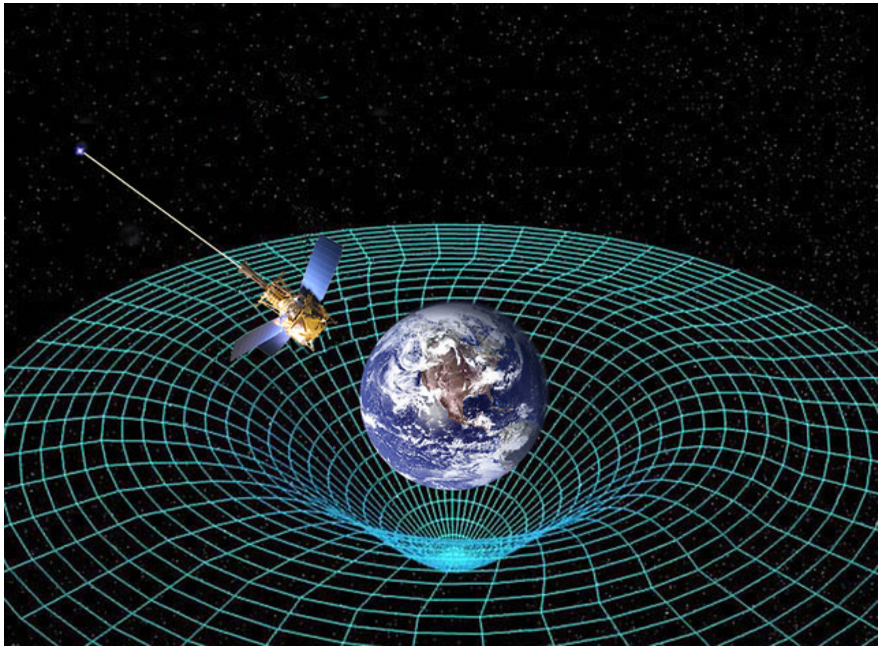

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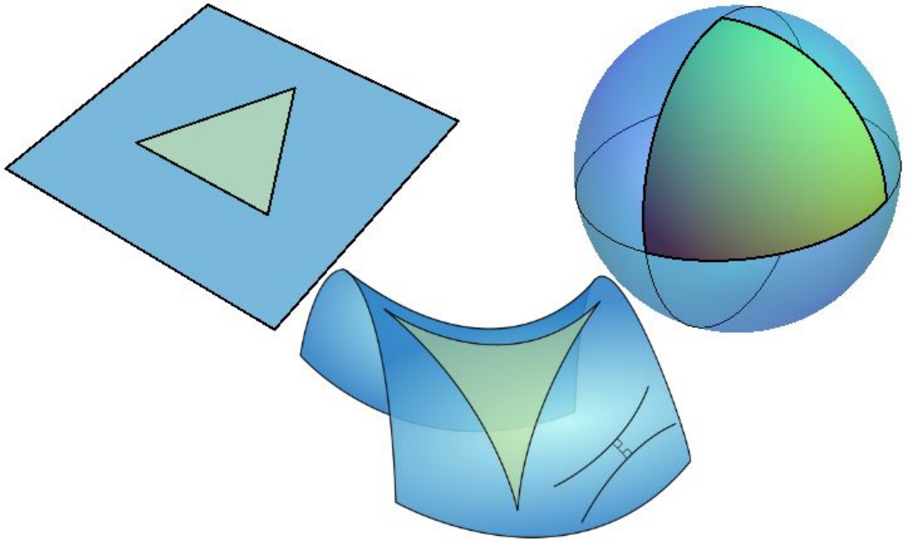

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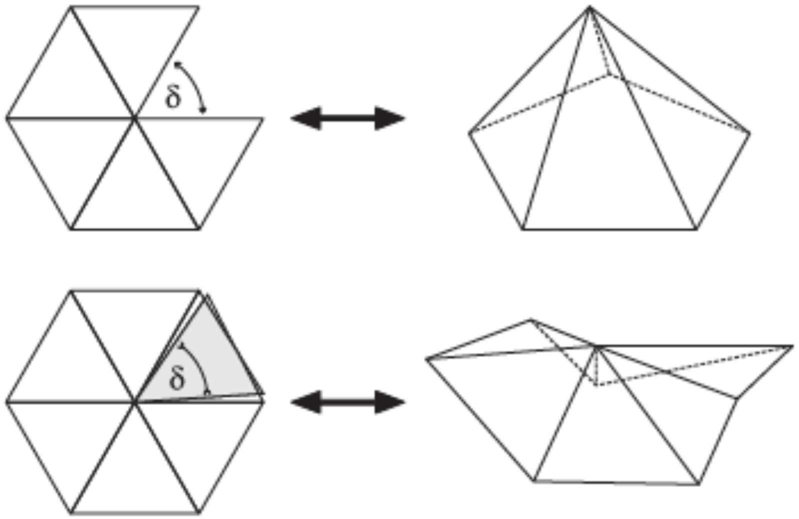

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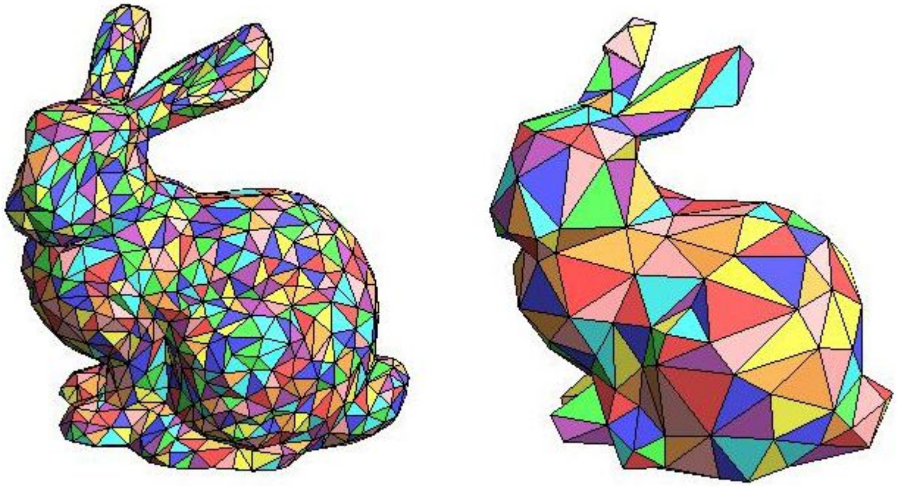

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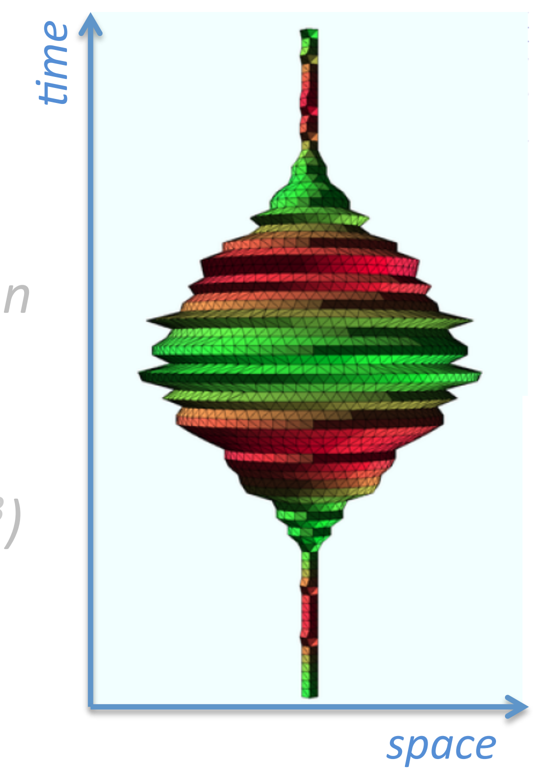

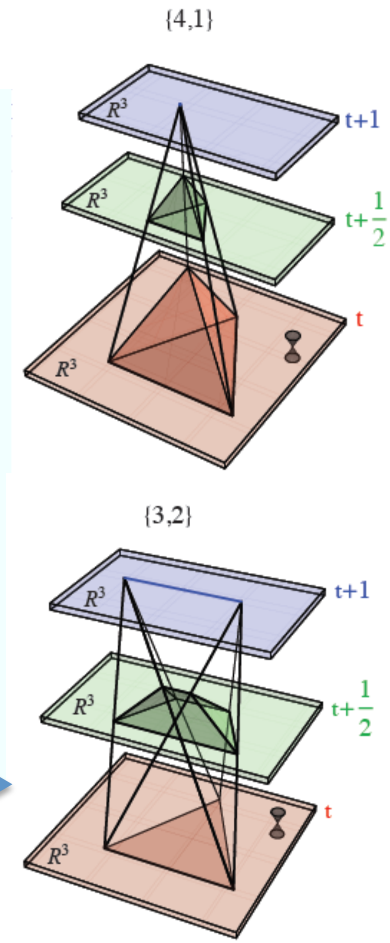

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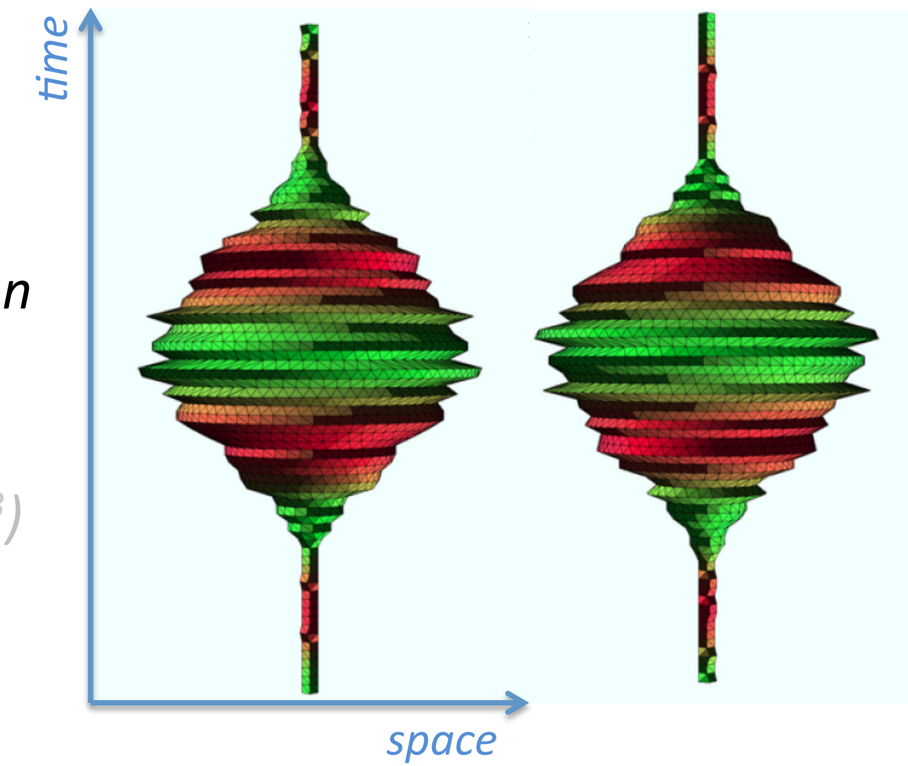

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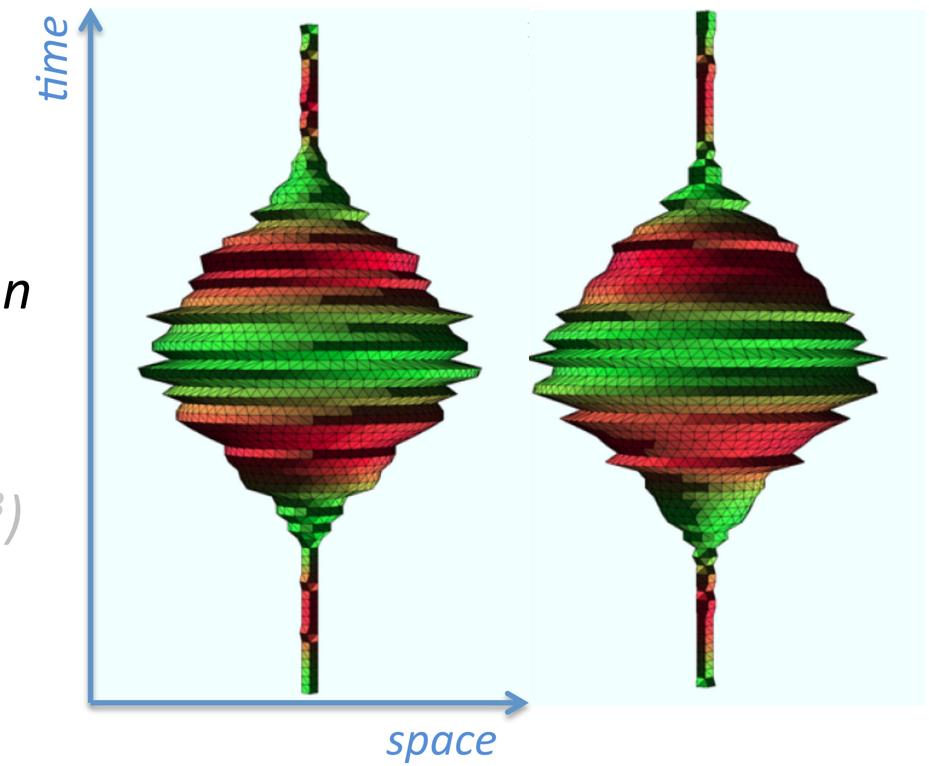

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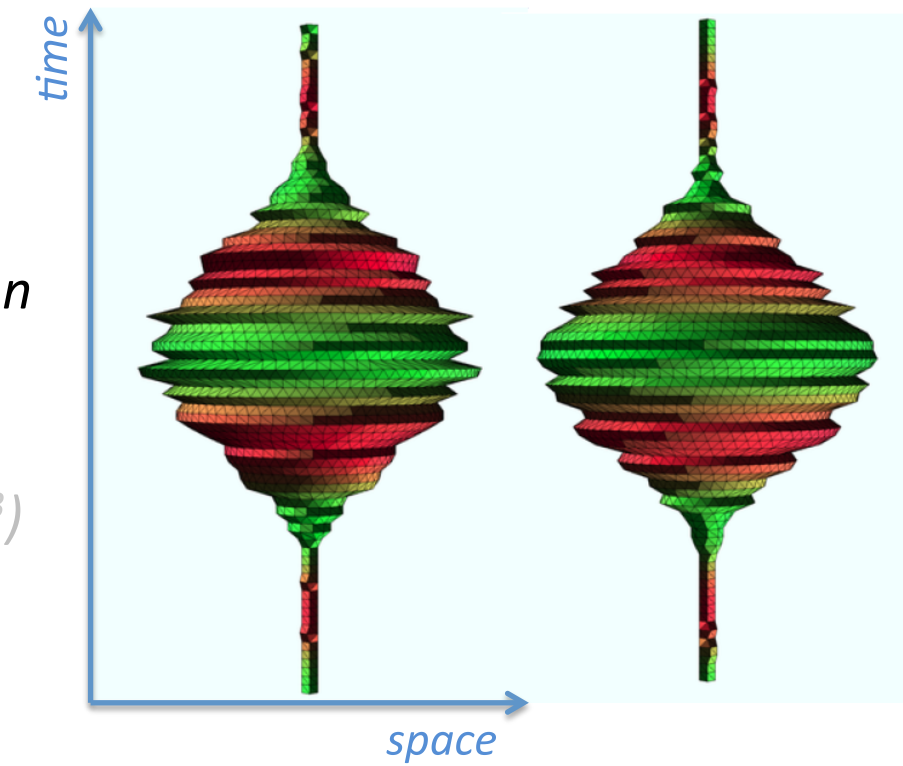

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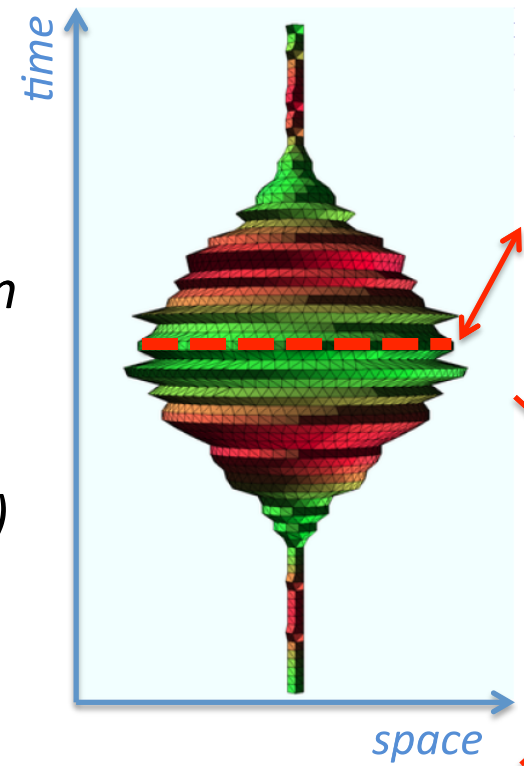

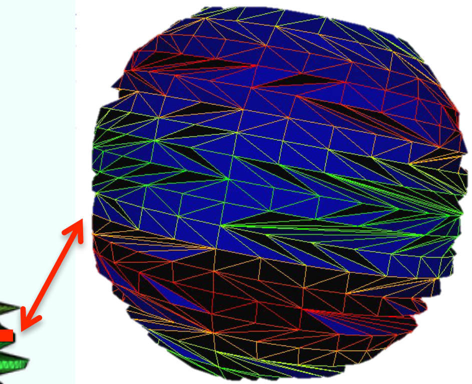

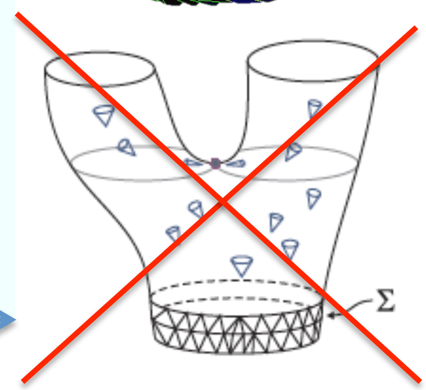

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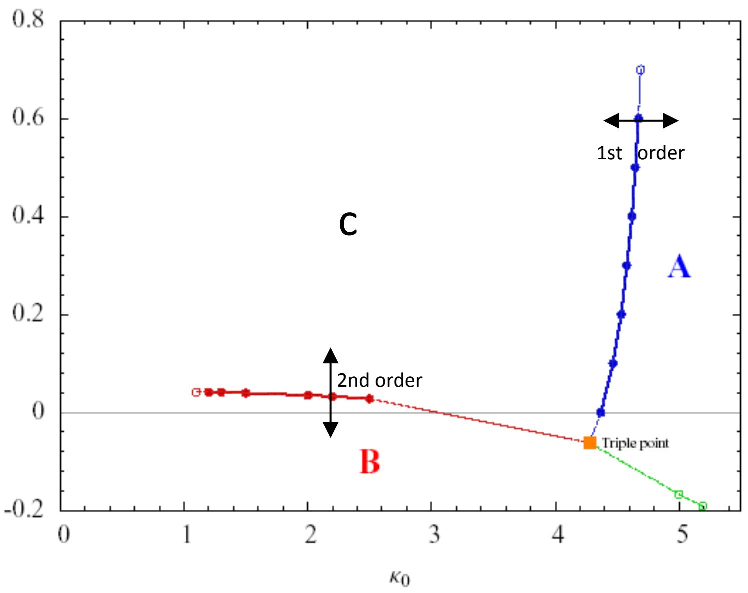

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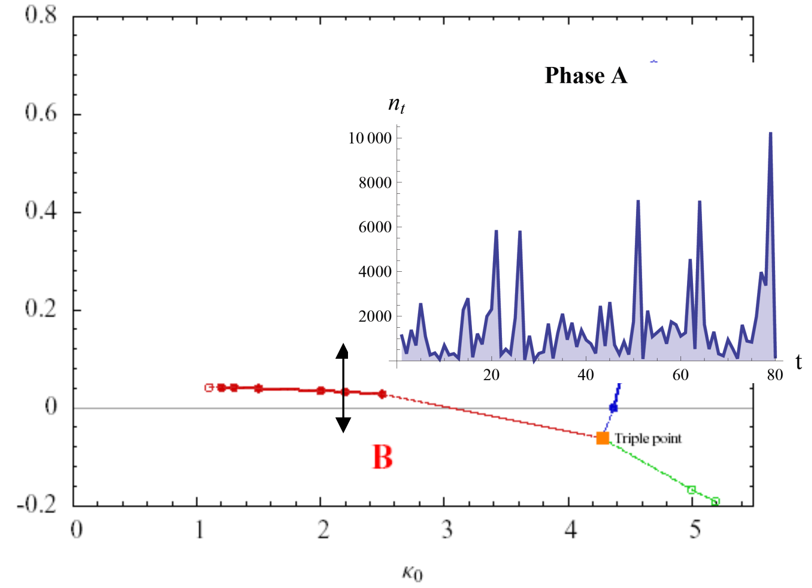

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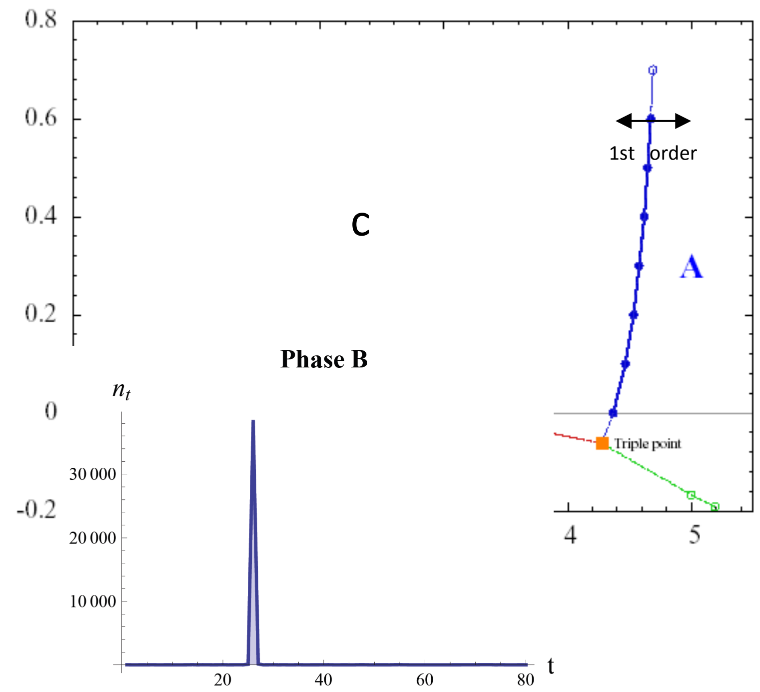

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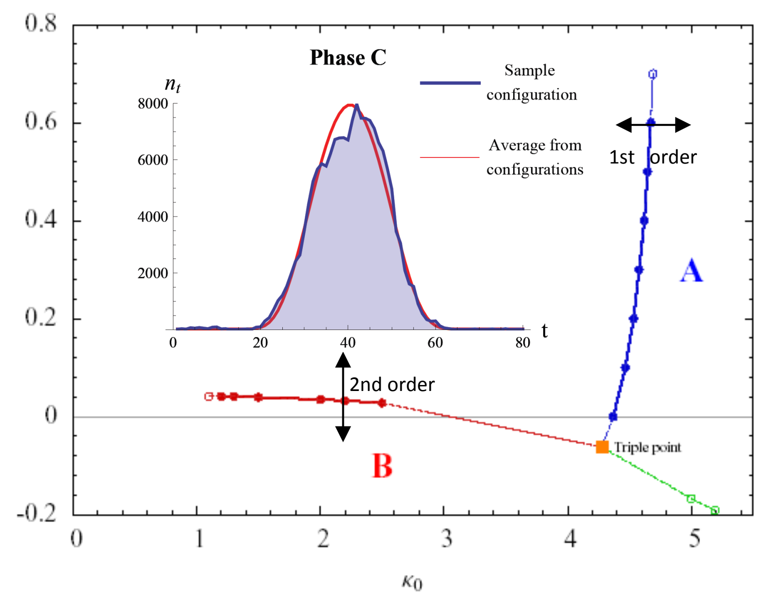

Effective action

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$$Z_{ef} = \sum_{\dots} \exp(-S_{ef}[\dots]) \quad \Longleftarrow \quad Z = \sum_{T} \exp(-S_R[T])$$

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- *Effective action describes quantum fluctuations of an observable (after „integrating out" other degrees of freedom)*
  - *Background geometry emerges dynamically: subtle interplay between bare action (S_R) and entropy of states*
  - *The observable: 3-volume of spatial layers (foliation leaves of the global proper time): $V_3(t)$*
  - *It is proportional to a number of (4,1) simplices, whose 3D faces (tetrahedra) form a given layer: $V_3(t) \propto n_t$*

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$$\updownarrow$$

$$S_e = k \ln \Omega$$

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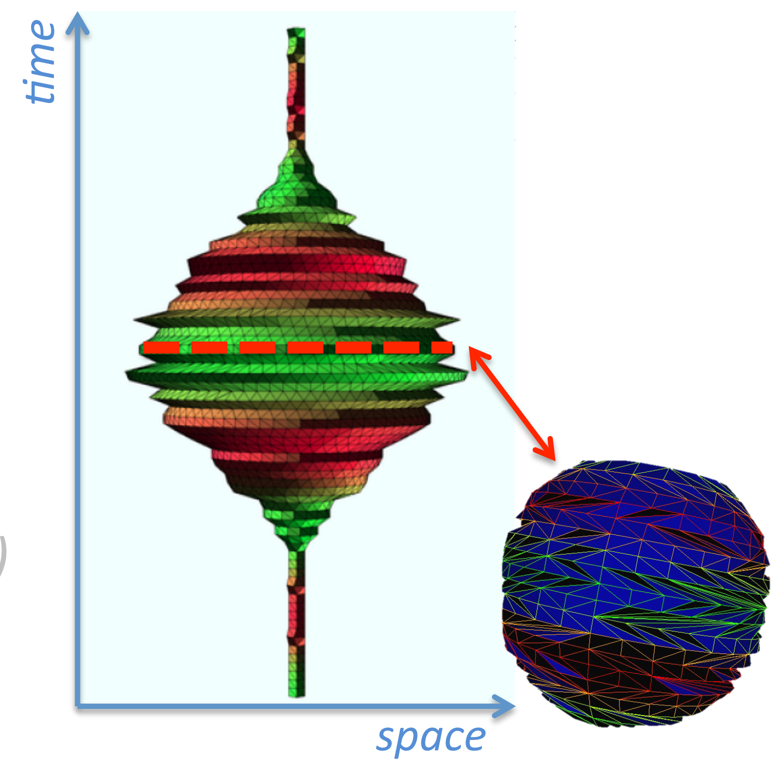

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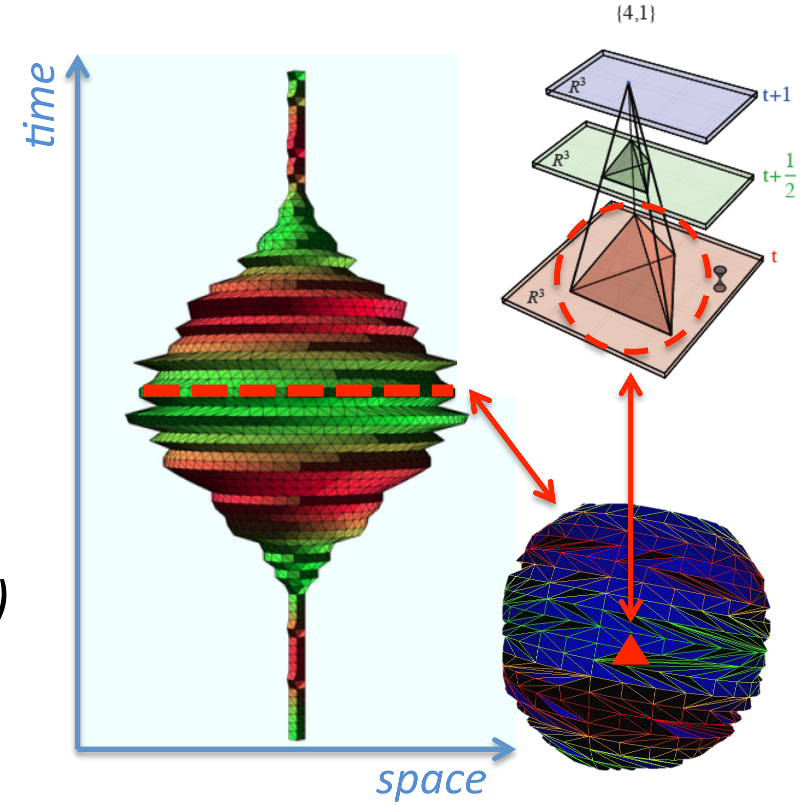

Effective action in Phase C

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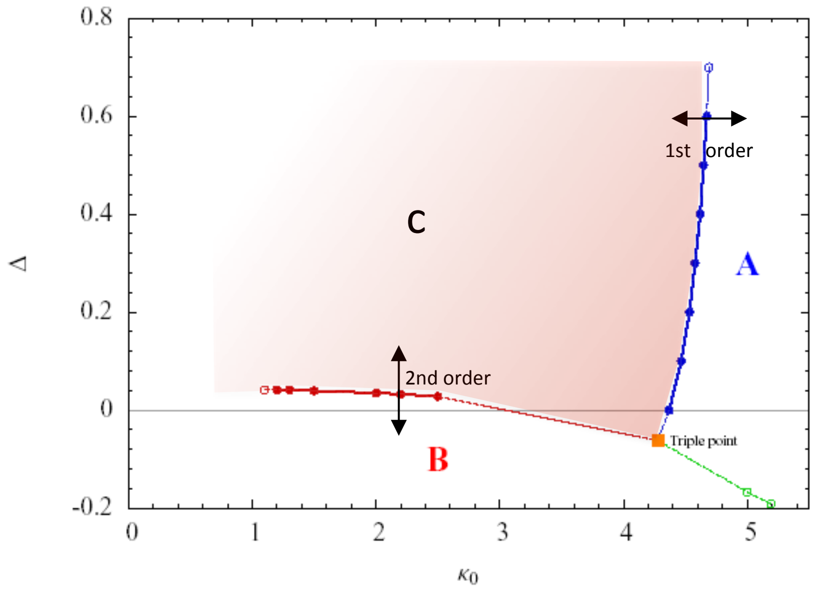

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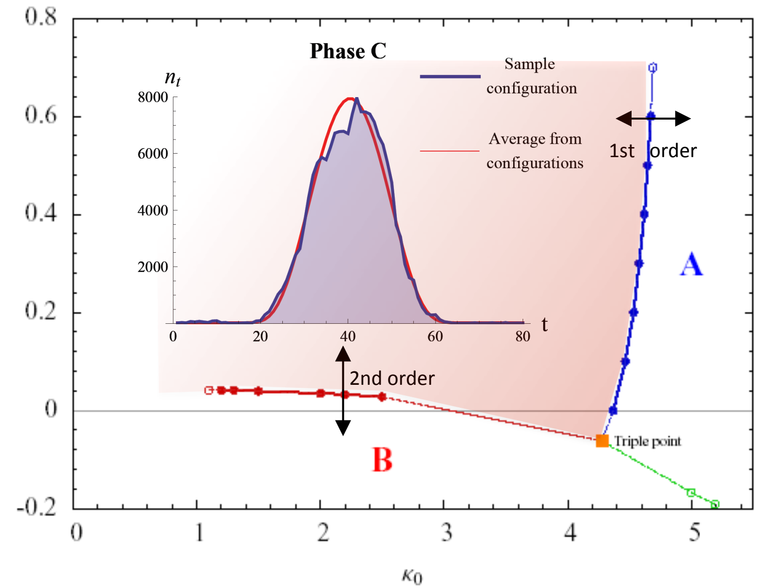
- ✧ *Background geommetry $<n_t>$ is consistent with a 4-dim sphere ⇒* Euclidean de Sitter universe (GR with positive cosmological constant)
- ✧ *This is clasically obtained for a homogenous and isotropic metric*
- ✧ *For which the GR action takes a form of the minisuperspace action*



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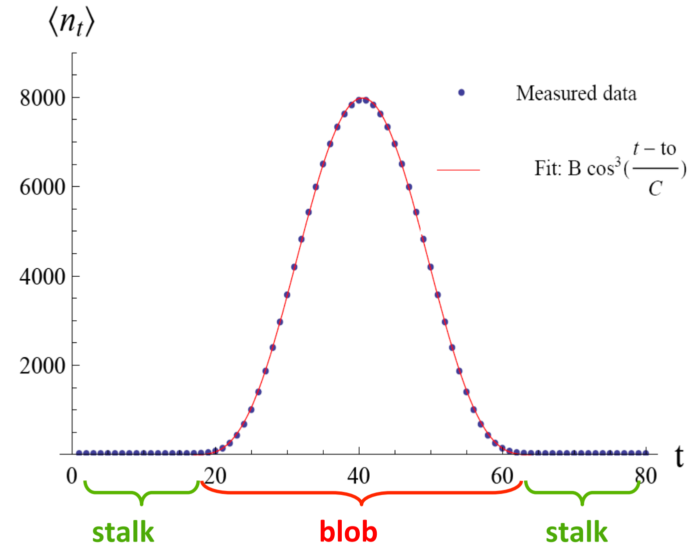

$$\bar{n}_t \equiv \langle n_t \rangle = \frac{3}{4}\tilde{V}_4 \frac{1}{\tilde{A}\tilde{V}_4^{1/4}} \cos^3\left(\frac{t-t_0}{\tilde{A}\tilde{V}_4^{1/4}}\right)$$

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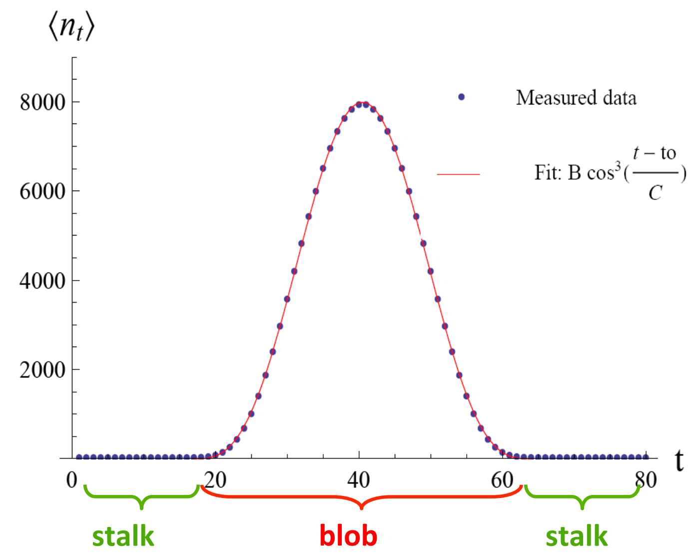

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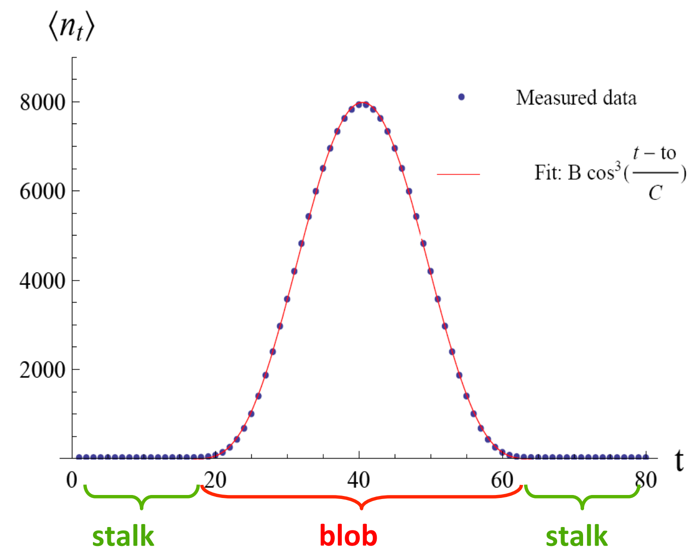

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✧ *CDT conjecture: the effective action in Phase C is a discretization of the minisuperspace action?*

$$S_{ef} = \frac{1}{\Gamma}\sum_t \left(\frac{\left(n_{t+1} - n_t\right)^2}{(n_t + n_{t+1})} + \tilde{\mu} n_t^{1/3} - \tilde{\lambda} n_t \right)$$

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- CDT conjecture: the effective action in Phase C is a discretization of the minisuperspace action?
  - The effective action can be analyzed by looking at quantum fluctuations around the semi-classical solution
  - The (inverse of) covariance matrix P =C-1 provides information about second derivatives of the effective action
  - The measured covariance matrix is consistent with MS action (with reversed overall sign) !

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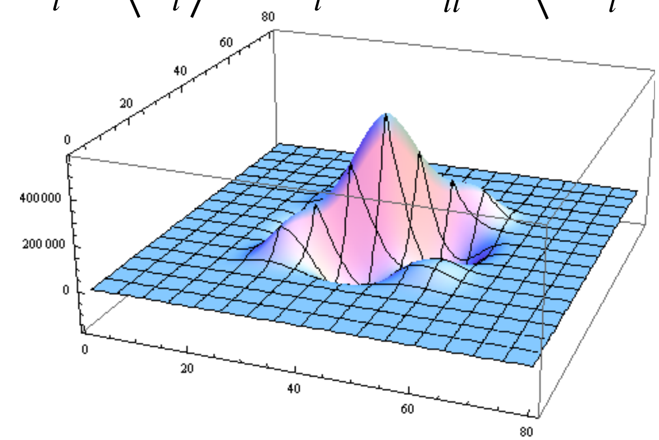

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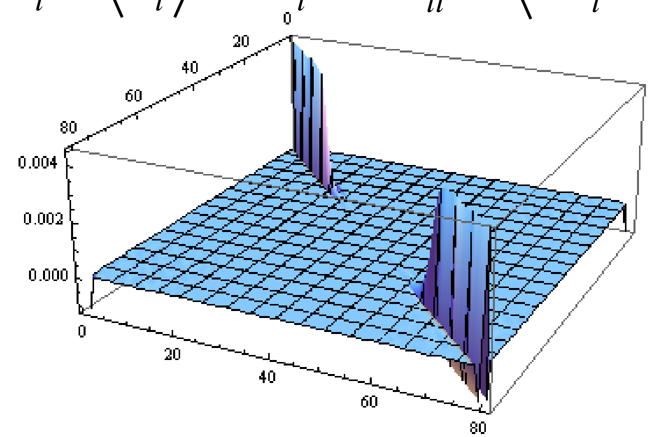

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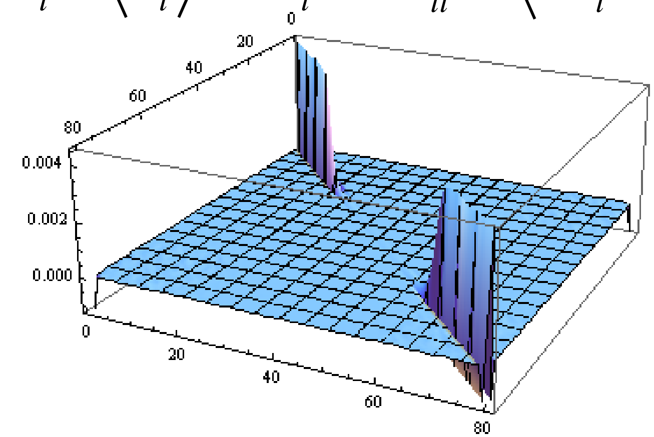

$$S_{ef} = \frac{1}{\Gamma} \sum_t \left(\frac{(n_{t+1} - n_t)^2}{(n_t + n_{t+1})} + \tilde{\mu} n_t^{1/3} - \tilde{\lambda} n_t \right)$$

$$S = -\frac{1}{24\pi G} \int dt \left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu V_3(t)^{1/3} - \lambda V_3(t) \right)$$

Transfer matrix method

✧ *The transfer matrix method enables to measure the effective action directly*

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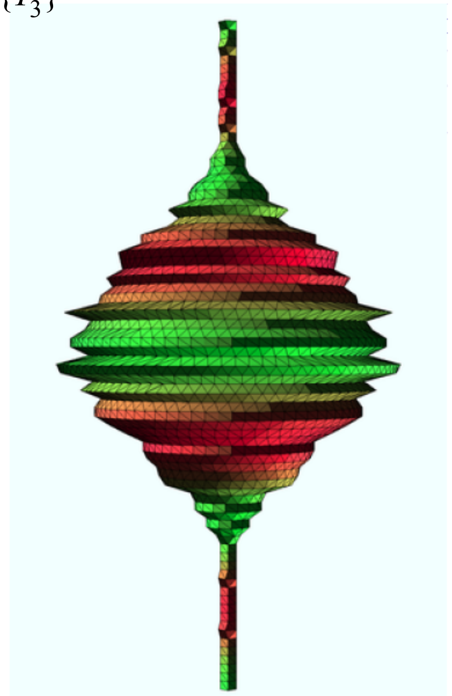
  - CDT has by definition a transfer matrix parametrized by 3-dimensional spatial triangulations T_3
  - Local form of the effective action in Phase C suggests that a description by effective transfer matrix parametrized by spatial volume n_t is also viable
  - Measurement of the transfer matrix = direct measurement of the effective Lagrangian

$$Z = \sum_{\{T_3\}} \langle T_3 | M^T | T_3 \rangle = tr M^T$$



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$$\left\langle n_t \,|\, M_{ef} \,|\, n_{t+1} \right\rangle \propto \exp\left(-L_{ef}[n_t, n_{t+1}]\right)$$

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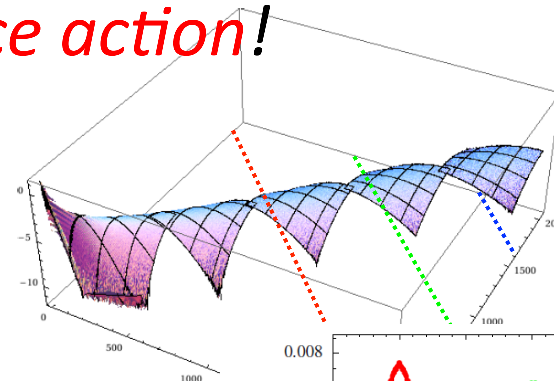

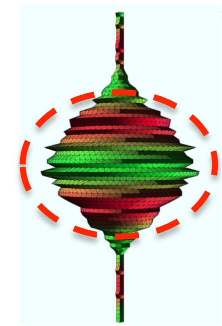

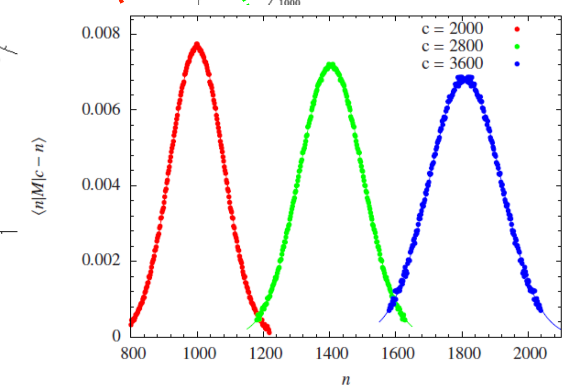

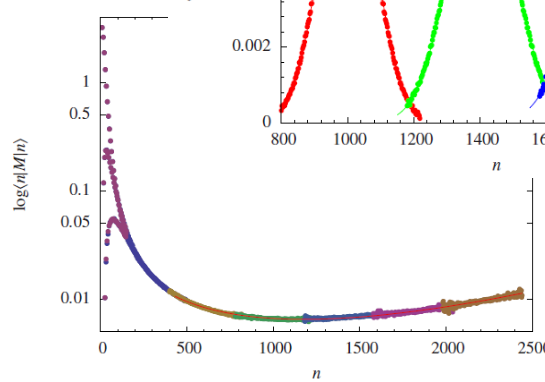

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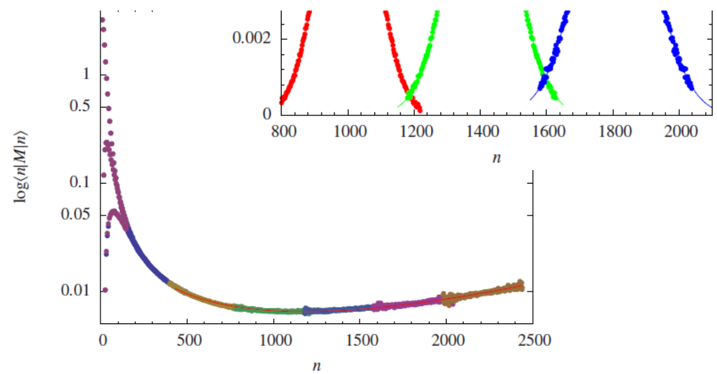
Method	Γ	n_0	μ	λ
Cross-diagonals	26.07 ± 0.02	-3 ± 1	–	–
Diagonal	(26.07)	–	16.5 ± 0.2	0.049 ± 0.001
Full fit	26.17 ± 0.01	7 ± 1	15.0 ± 0.1	0.046 ± 0.001
Covariance matrix *	26.5 ± 1.0	–	20 ± 2	–



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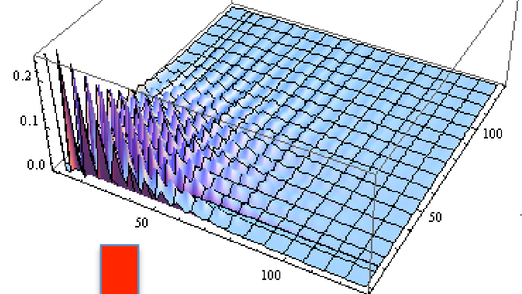

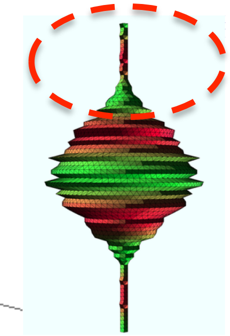

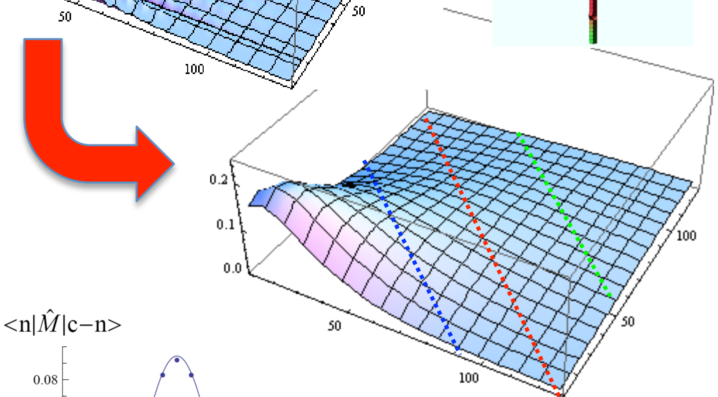

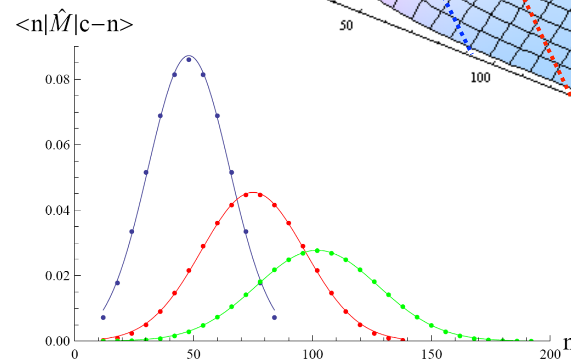

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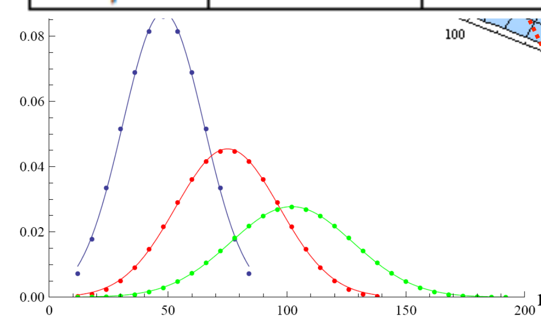
small vol. correction

$$L_{ef} = \frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\left(\frac{n+m}{2}\right) + \delta\left(\frac{n+m}{2}\right)^{-\rho}\right]$$

Parameter	Stalk	Blob
Γ	27.2 ± 0.1	$25.7 - 26.2$
n_0	5 ± 1	$-3 - +7$
μ	34 ± 2	$13 - 30$
λ	0.12 ± 0.02	$0.04 - 0.07$
δ	$(4 \pm 7) \times 10^4$	–
ρ	3 ± 1	–



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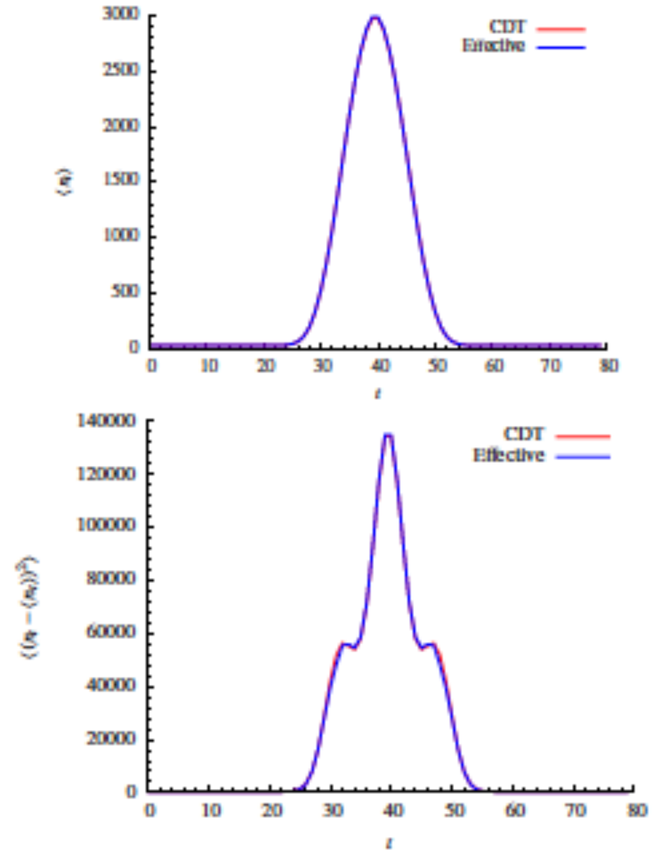

Effective action in Phases A & B

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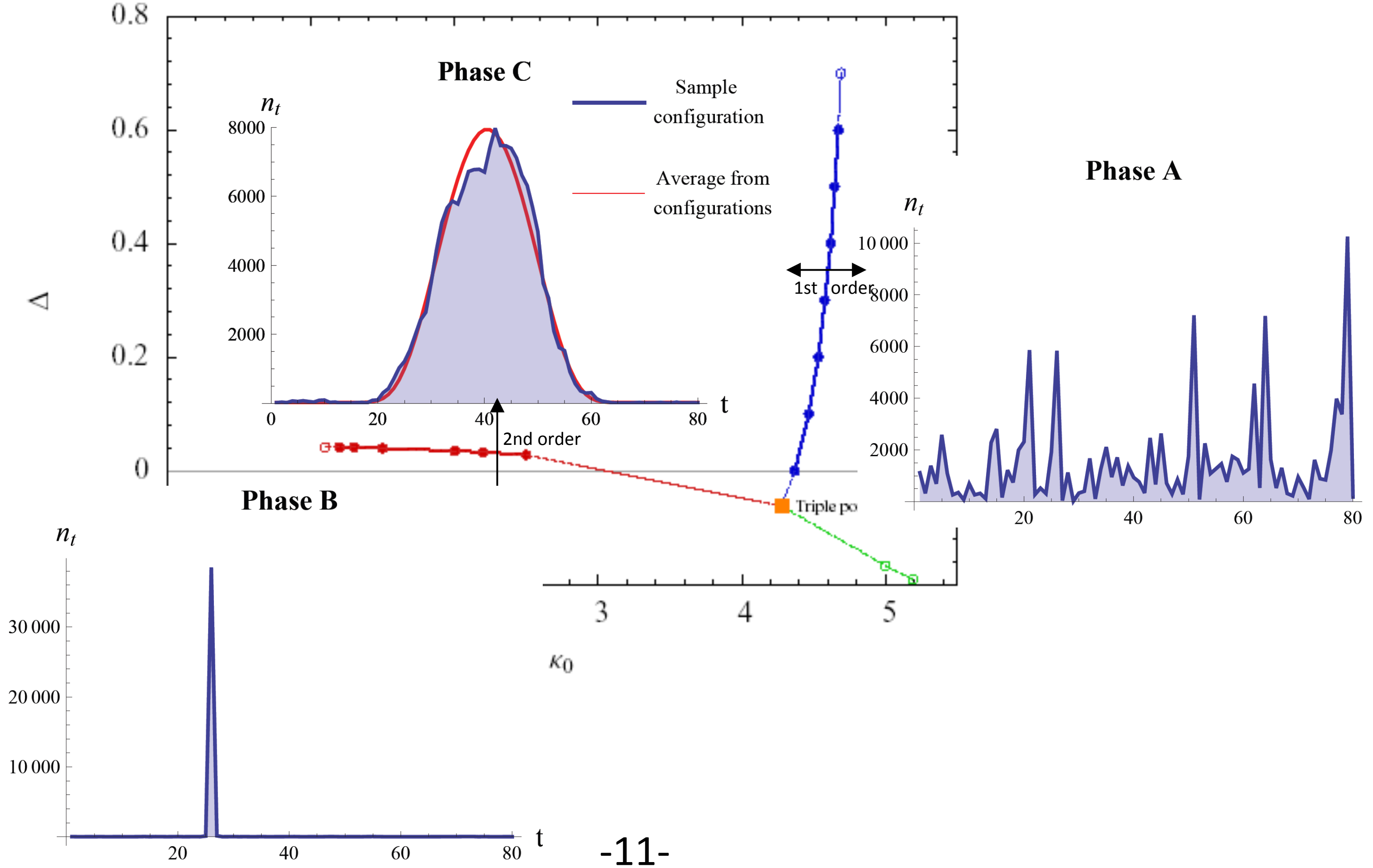

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- *The transfer matrix can be used to determine the form of the effective action in other phases …*
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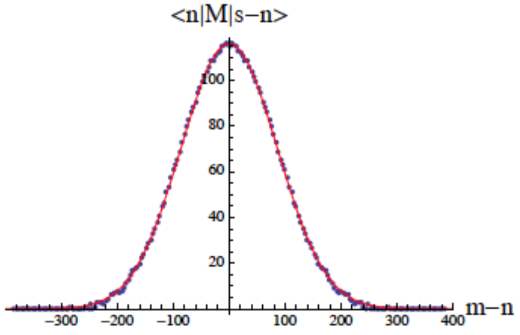
$$L_C = \frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\left(\frac{n+m}{2}\right)\right]$$



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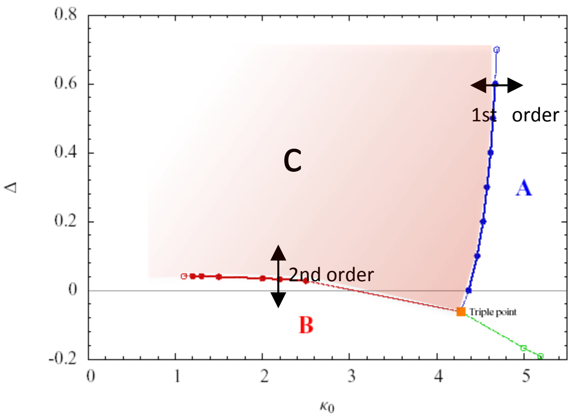

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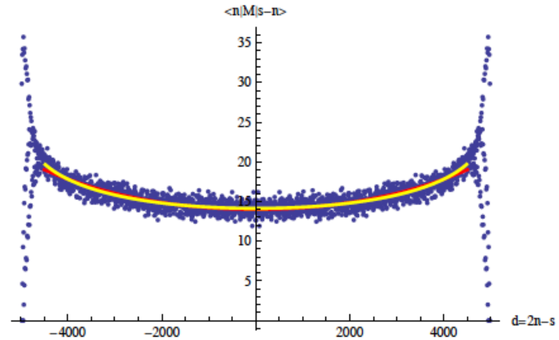

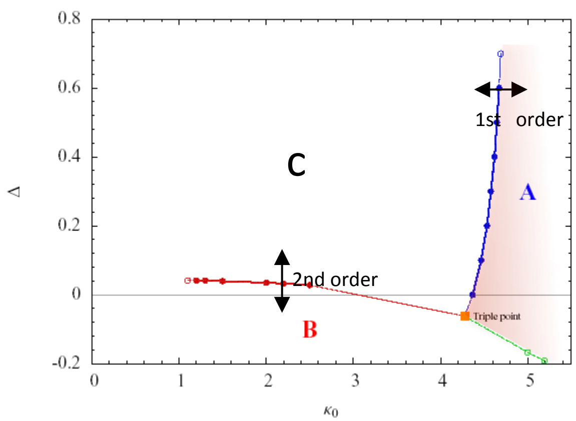

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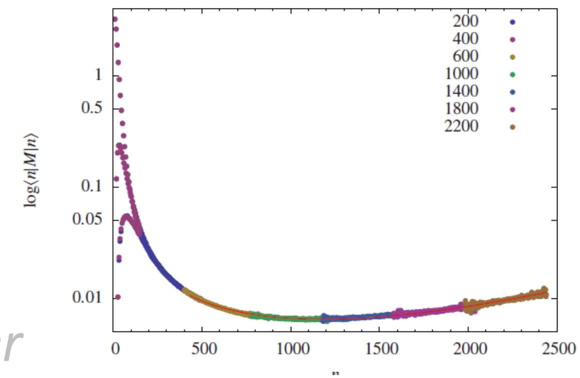

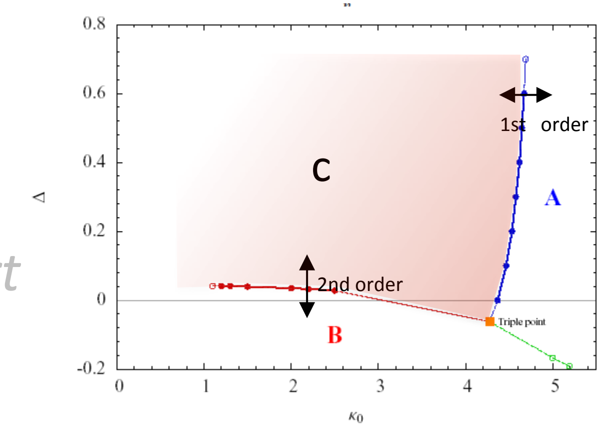

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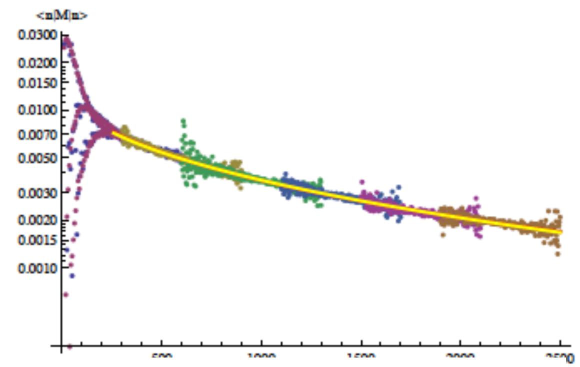
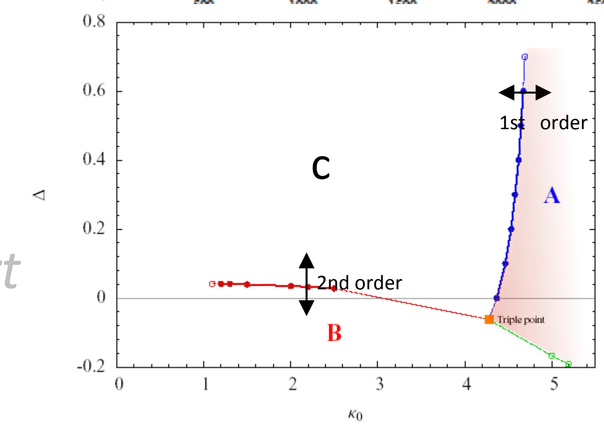
$\alpha \neq 1/3$

$$L_A = \mu(n^{\alpha} + m^{\alpha}) + \lambda(n+m)$$





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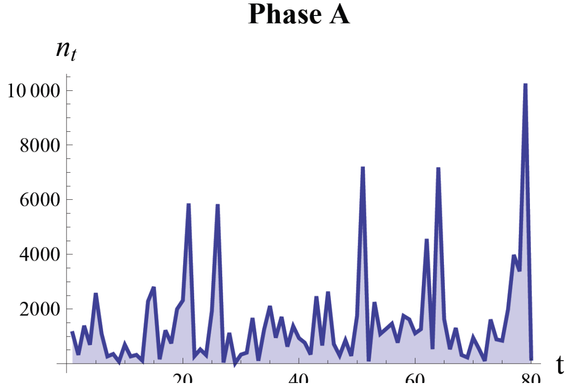

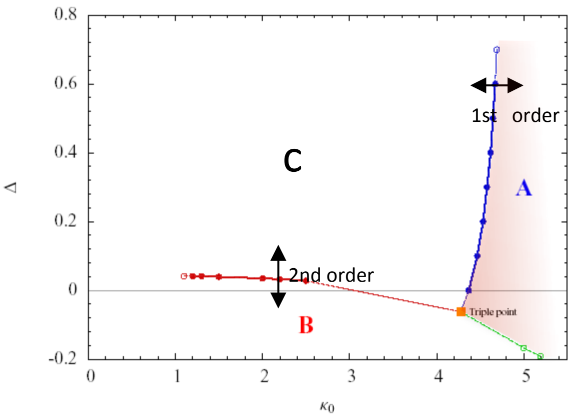

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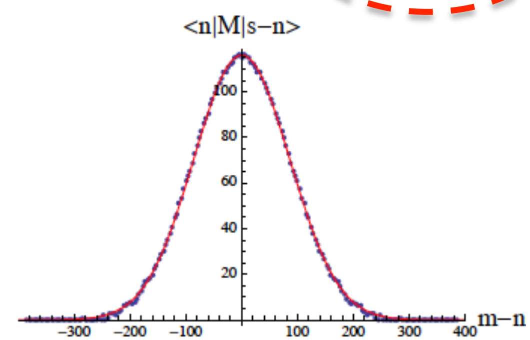

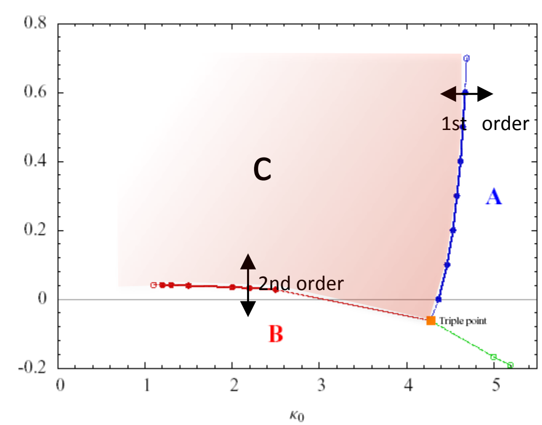

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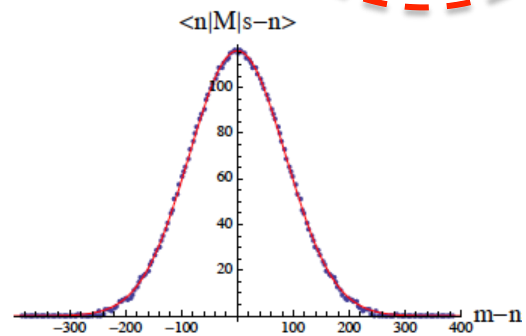

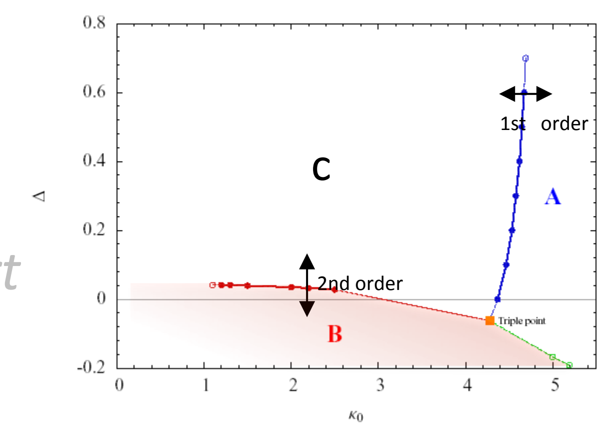

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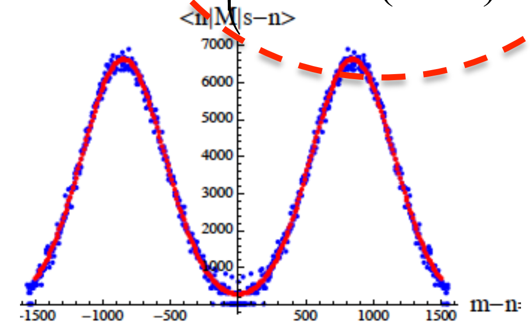

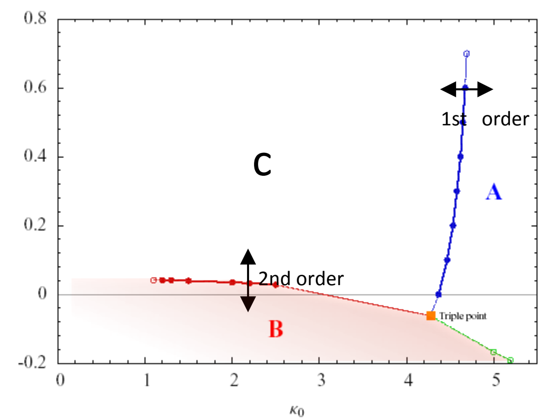

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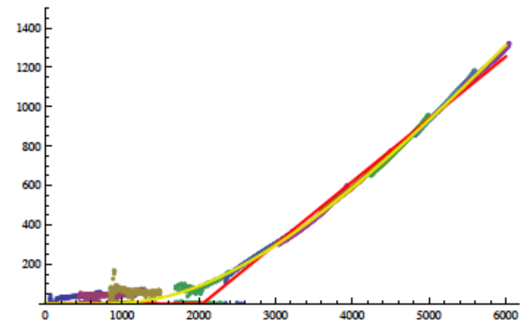

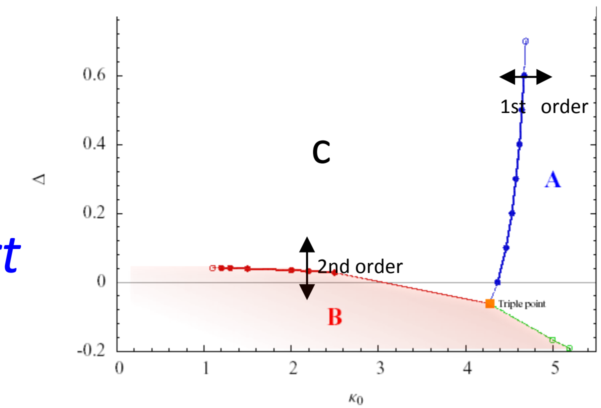

Phase transitions

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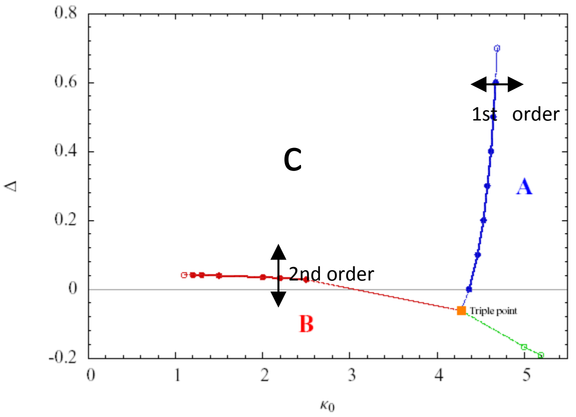

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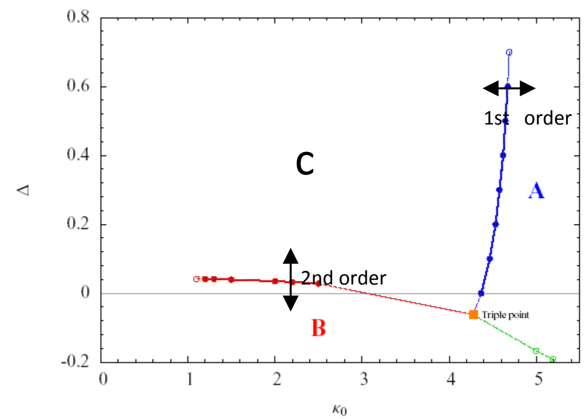

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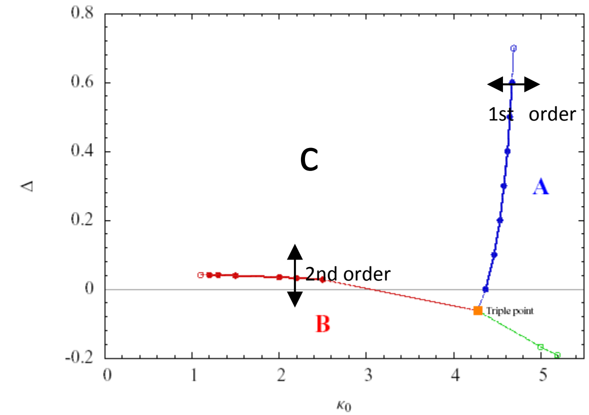

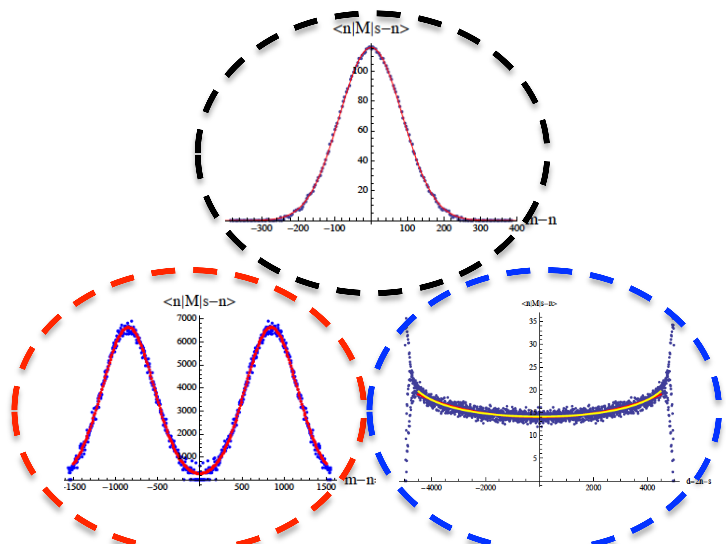

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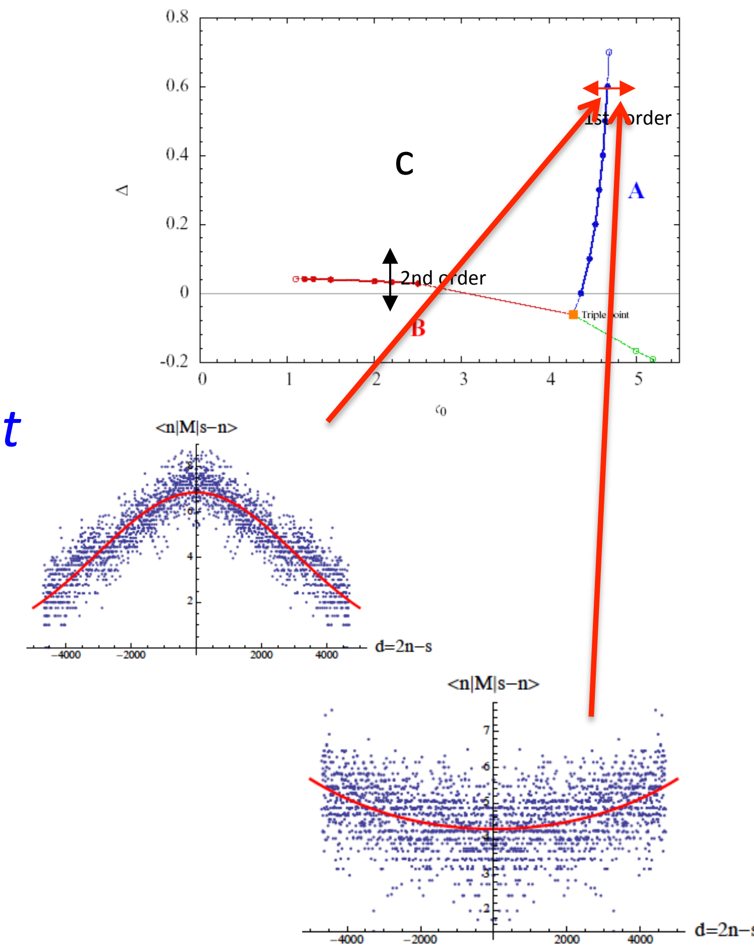

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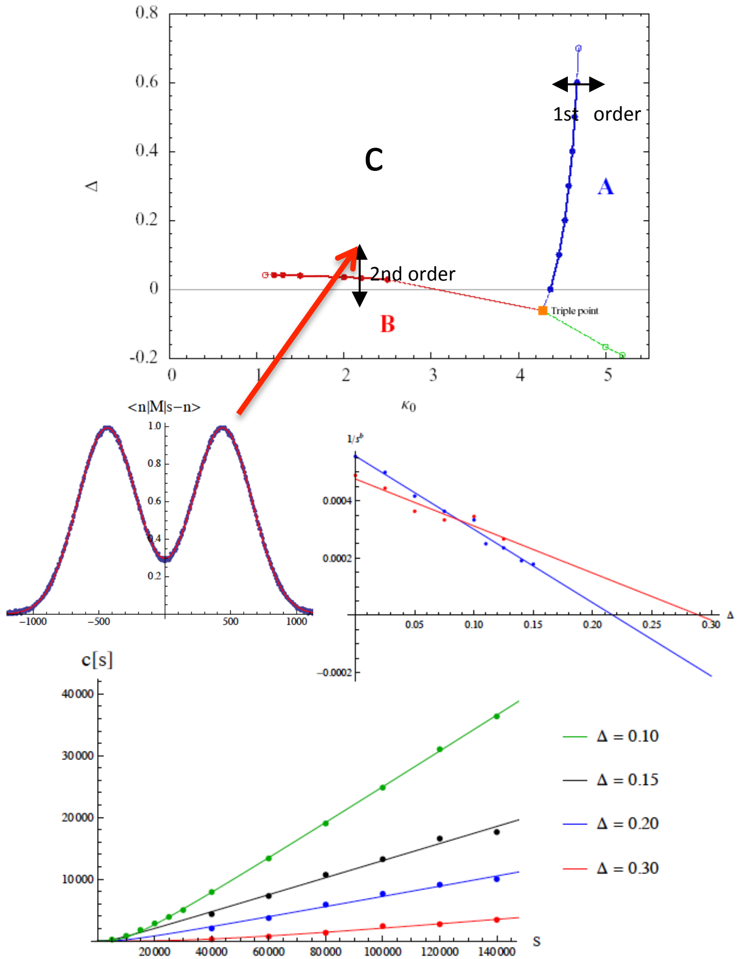

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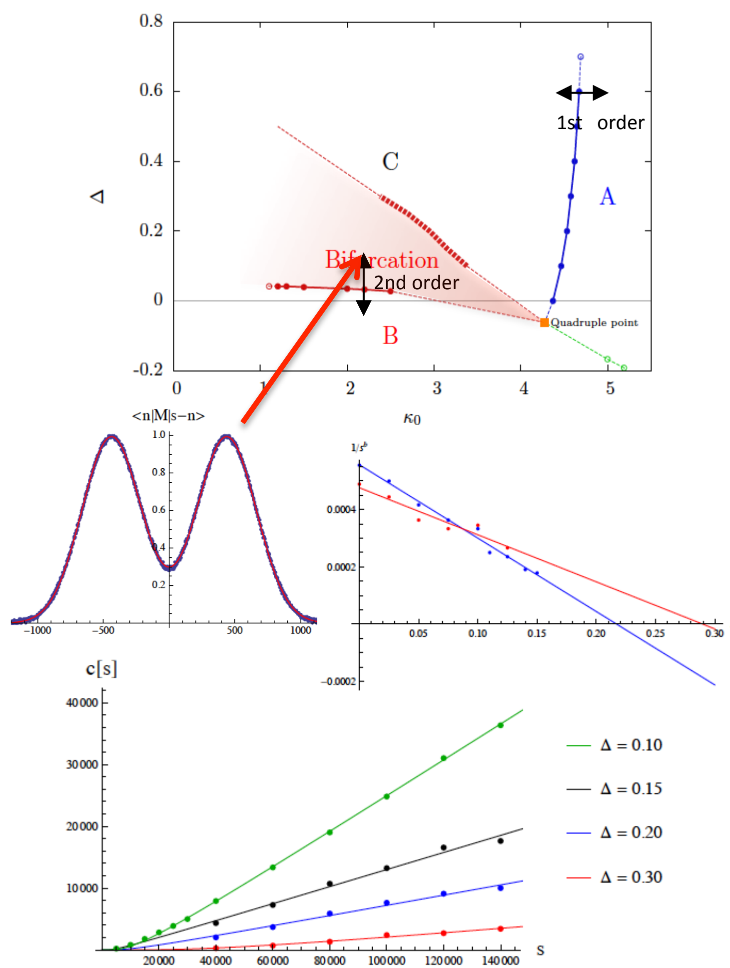
✧ *The A ⇔ C phase transition is consistent with a change of the effective action*

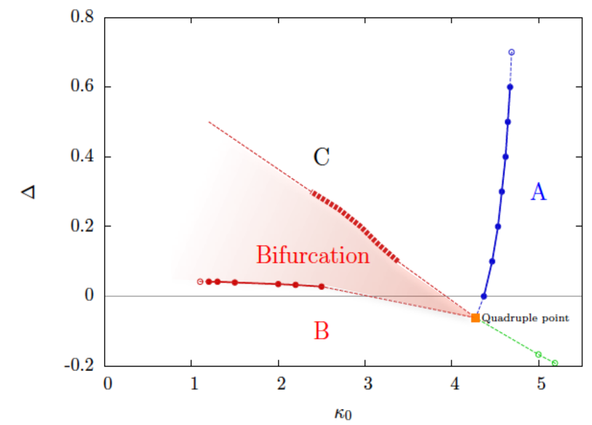
✧ *The bifurcation of the kinetic part observed in Phase B persists in some region of parameter space originally denoted Phase C*

✧ *A new „bifurcation" phase exists between Phases B & C*



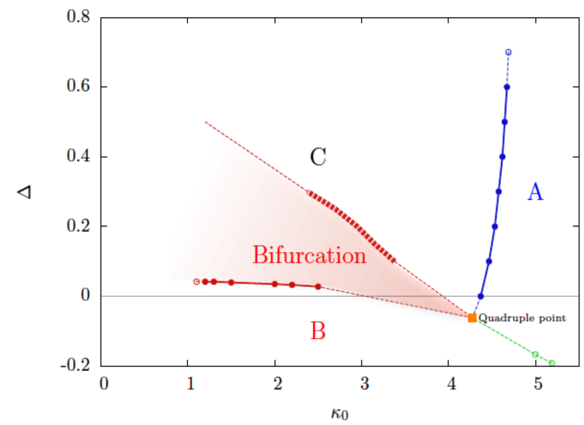
Bifurcation phase

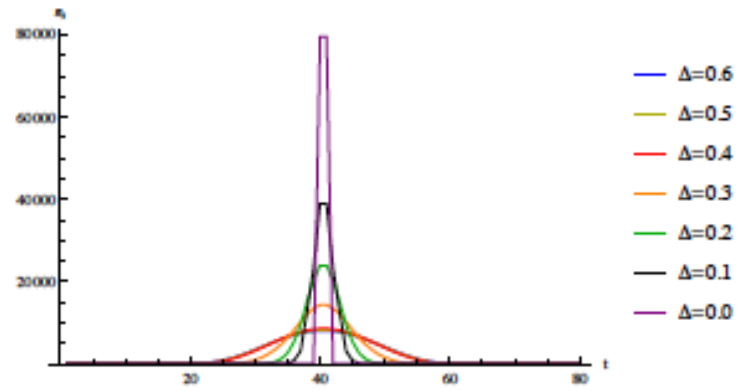
✧ *The new phase separating phases B & C is related to a bifurcation of the effective action ..*



Bifurcation phase

- *The new phase separating phases B & C is related to a bifurcation of the effective action ..*
  - *Average volume profile in the new phase resembles the profile observed in Phase C …*
  - *… but the profile is shrinking in time direction …*
  - *… which is well explained by the bifurcation of the transfer matrix kinetic term*





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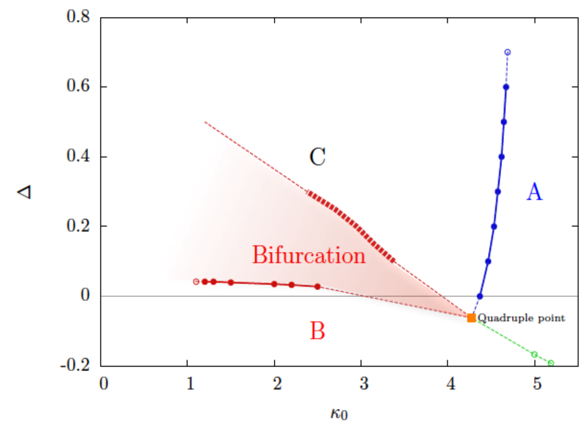

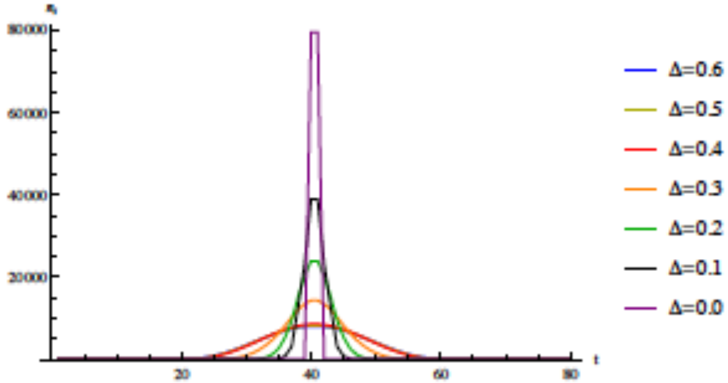

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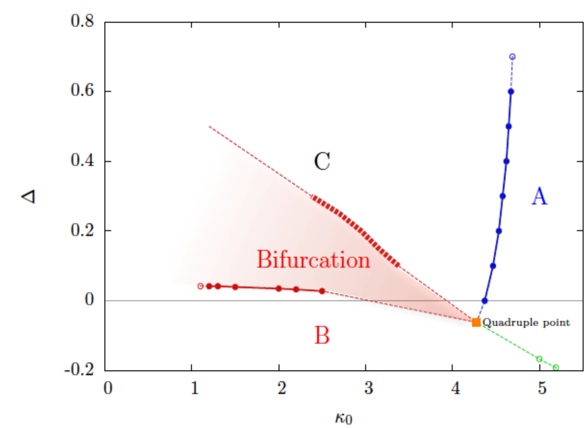

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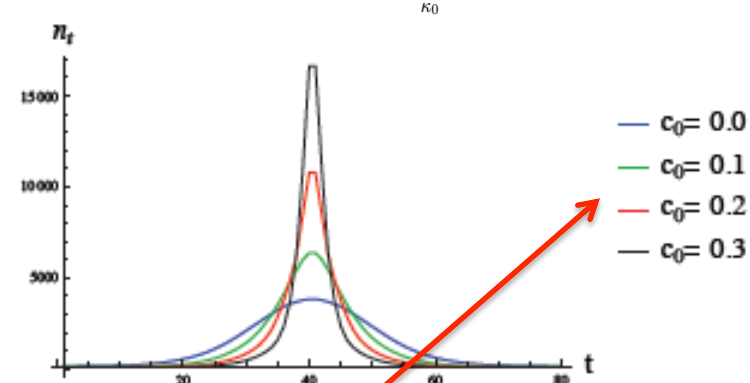
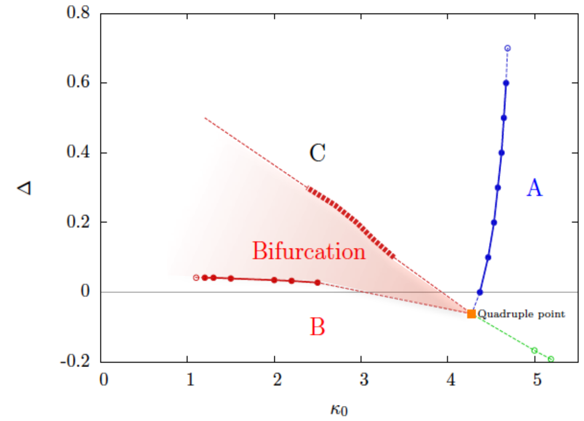


$$\langle n|M_B|m\rangle = N[n+m]\left[\exp\left(-\frac{\left(m-n-\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m)}\right)+\exp\left(-\frac{\left(m-n+\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m)}\right)\right]$$

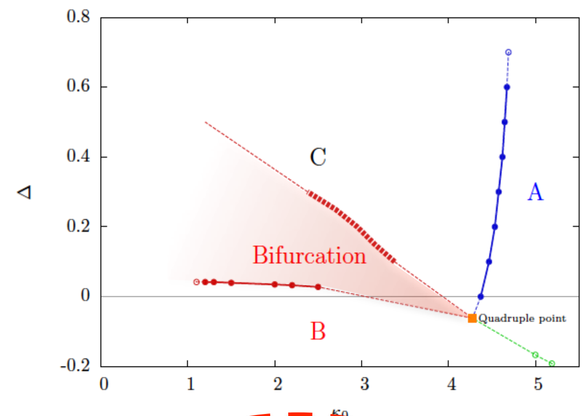
Bifurcation phase

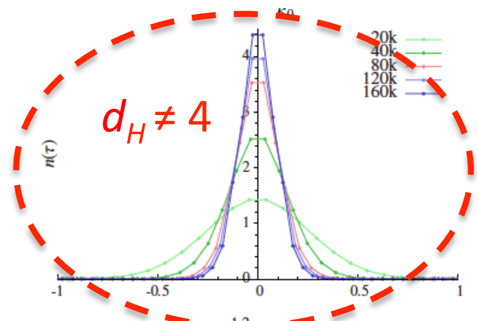
✧ … resulting from geometry considerably different than inside Phase C

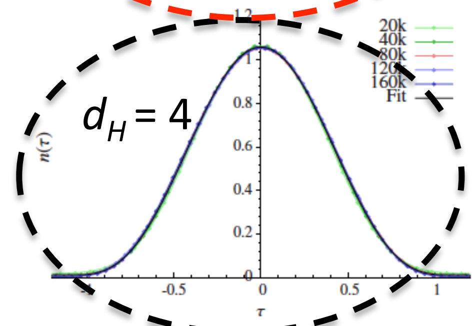


Bifurcation phase

- *... resulting from geometry considerably different than inside Phase C*
  - *Infinite Hausdorff dimension?*
  - *Spectral dimension > 4 and growing (to infinity ?) with growing volume*
  - *This suggests high connectivity between the building blocks*
  - *Spatial volume is concentrated in short geodesic distance*
  - *Such volume clusters appear every second time slice and are linked by „singular" vertices*
  - *Phase transition brakes (approximate) translational symmetry in space direction*







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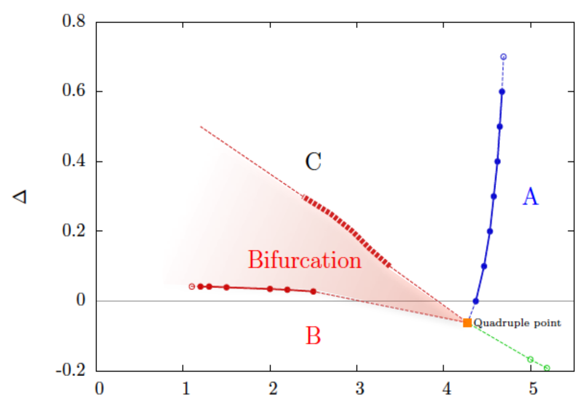

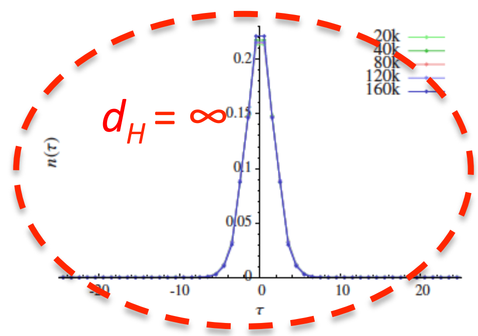

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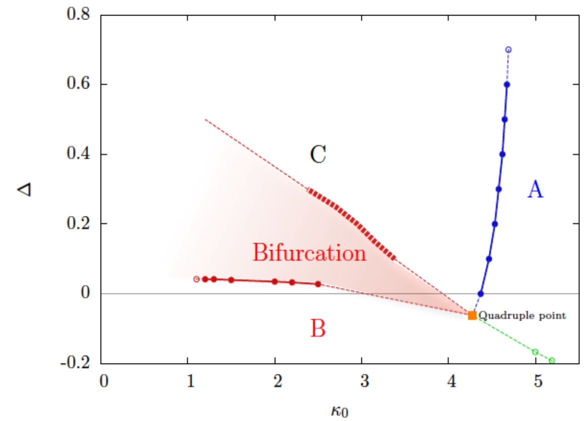

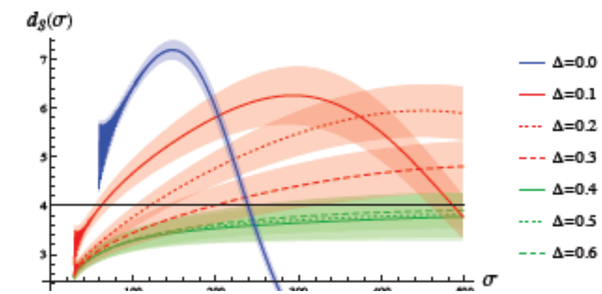

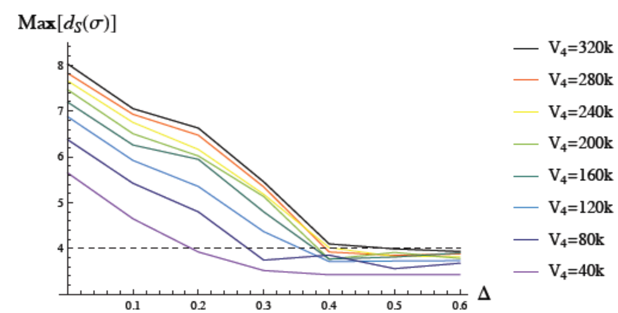

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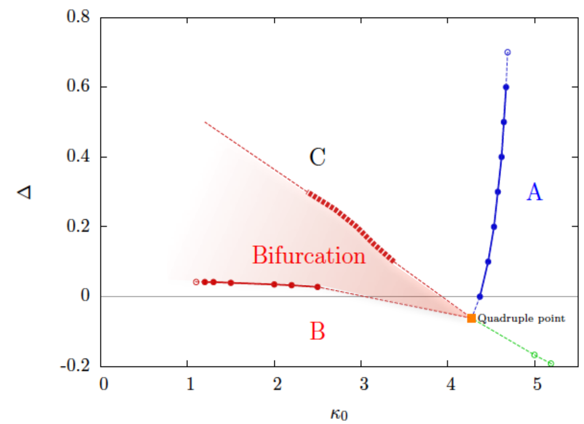

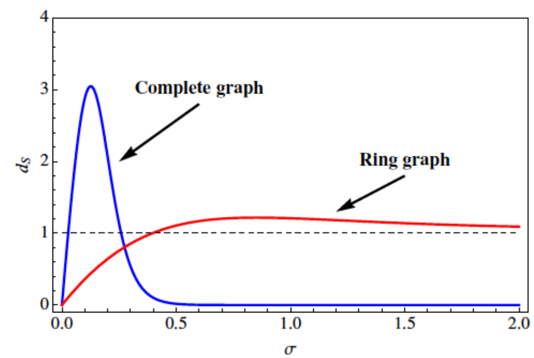

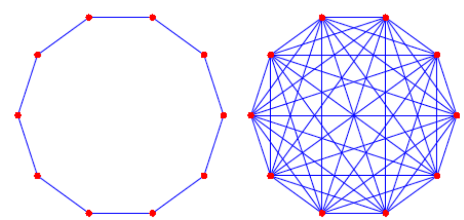

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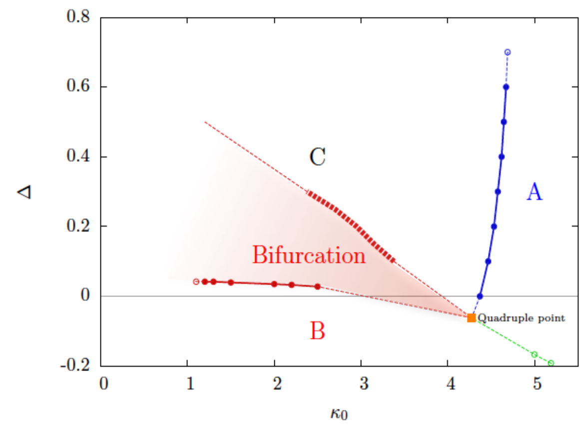

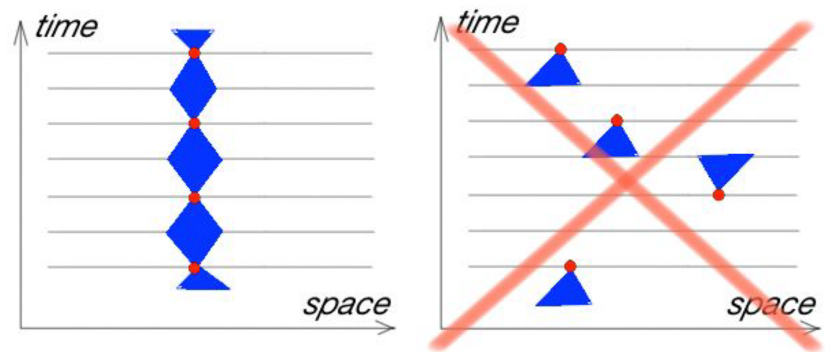

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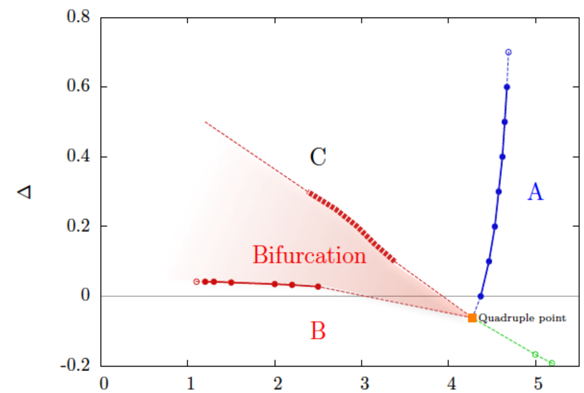
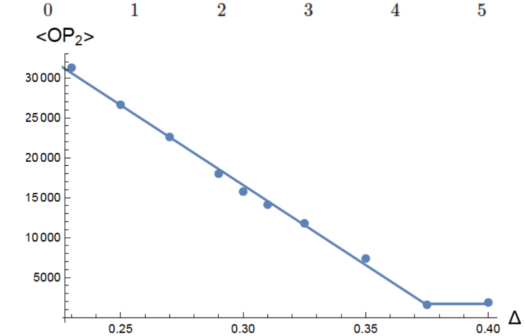
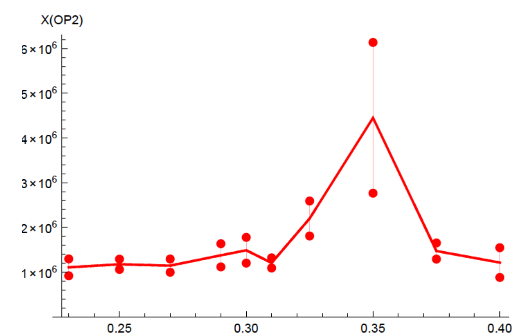
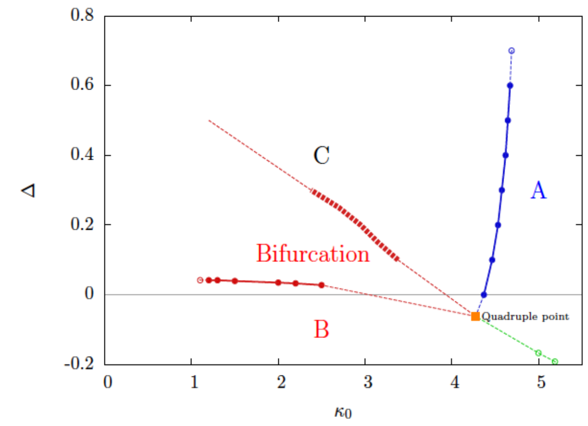






Signature change

✧ *Bifurcation of the effective action near phase transition can be interpretted as a spontaneous signature change*



Signature change

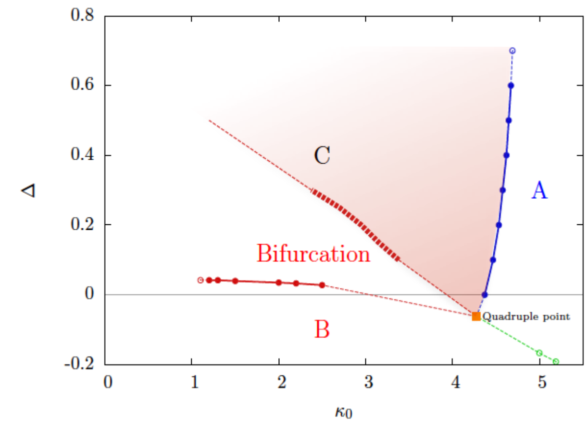
✧ Bifurcation of the effective action near phase transition can be interpretted as a spontaneous signature change

✧ The transfer matrix bifurcates at the new phase transition

✧ The phase transition is related to: $c_0 \rightarrow 0$ and $s_b \rightarrow \infty$ limit

✧ For large latice volumes ($n+m \rightarrow \infty$) one can expand in powers of $2c_0(n-m)/\Gamma << 1$

✧ The form of the effective Lagrangian can be viewed as a spontaneous Wick rotation of the metric ($t \rightarrow it$) compared to Phase C



$$\langle n | M_C | m \rangle = N[n+m] \exp\left(-\frac{(m-n)^2}{\Gamma(n+m)} \right)$$

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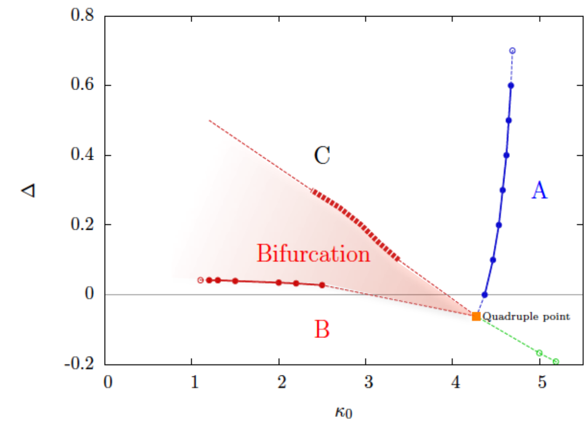

$$\langle n | M_B | m \rangle = N[n+m] \left[\exp\left(-\frac{\left(m-n-\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m)} \right) + \exp\left(-\frac{\left(m-n+\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m)} \right) \right]$$

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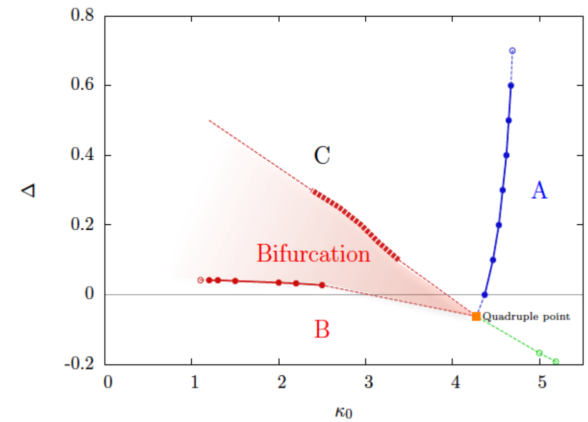

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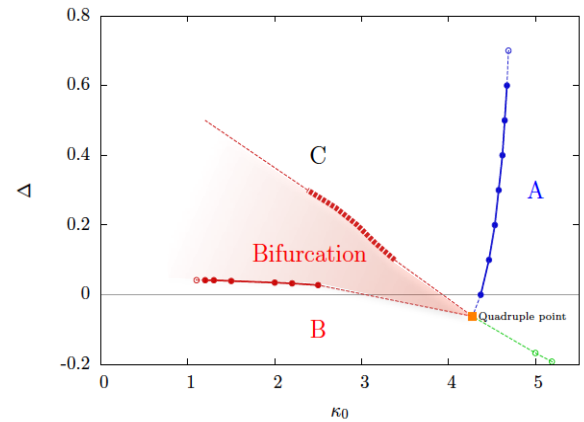

$$\langle n | M_B | m \rangle = N[n+m] \exp\left[-\frac{c_0^2}{\Gamma}(n+m)\right] \exp\left[-\left(1-\frac{2c_0^2(n+m)^2}{\Gamma}\right)\frac{1}{\Gamma}\frac{(m-n)^2}{(n+m)} - \frac{4}{3}\left(\frac{c_0(n-m)}{\Gamma}\right)^4 + \ldots\right]$$

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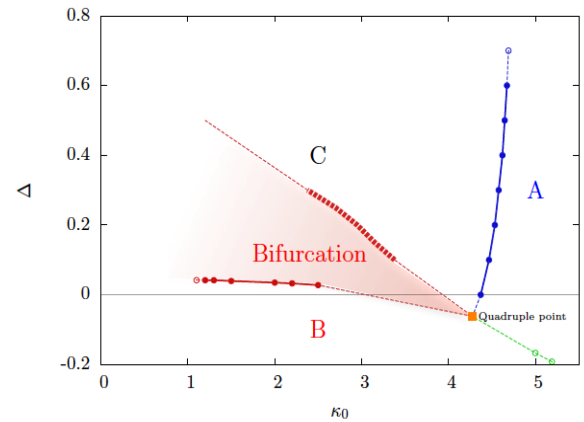

$$L_B = \left(1 - \frac{2c_0^2(n+m)^2}{\Gamma}\right)\frac{1}{\Gamma}\frac{(n-m)^2}{n+m} + potential[n+m]$$

$$\langle n | M_B | m \rangle = N[n+m]\exp\left[-\frac{c_0^2}{\Gamma}(n+m)\right]\exp\left[-\left(1 - \frac{2c_0^2(n+m)^2}{\Gamma}\right)\frac{1}{\Gamma}\frac{(m-n)^2}{(n+m)} - \frac{4}{3}\left(\frac{c_0(n-m)}{\Gamma}\right)^4 + \dots\right]$$

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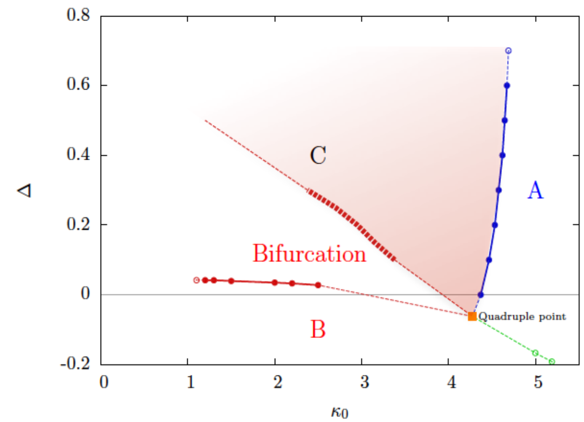

$$L_c = \frac{1}{\Gamma}\frac{(n-m)^2}{n+m} + potential[n+m]$$

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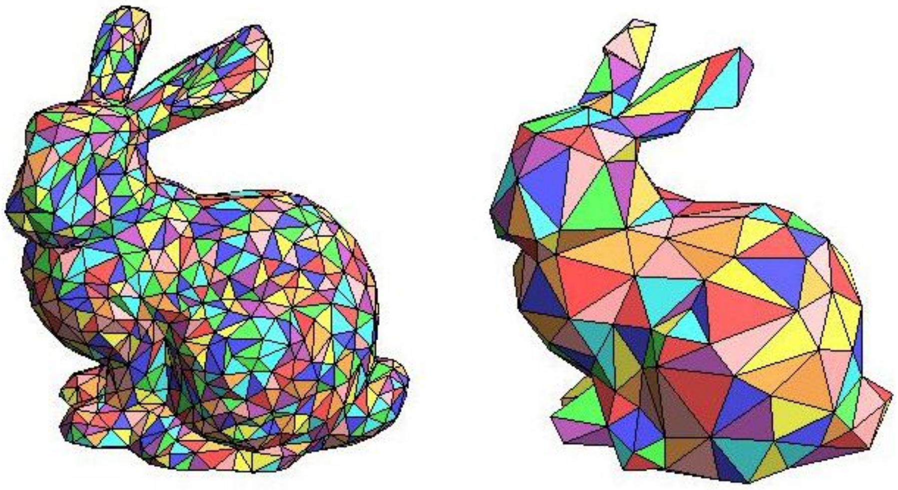
Conclusions

- ✧ *Transfer matrix approach allows one to measure the effective action directly*
- ✧ *The action inside Phase C is well described by the MS model*
- ✧ *The transfer matrix method gives access to effective action in other phases*
- ✧ *In Phase A the kinetic term vanishes* $\Rightarrow$ *possible relation to asymptotic silence ?*
- ✧ *In Phase B one observes a bifurcation of the kinetic term*
- ✧ *New Bifurcation Phase with nontrivial geometric properties was discovered*
- ✧ *New phase transition might be related to signature change*



Thank You !

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