What is Dimensional Reduction Really Telling Us?

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Outline

1. Introduction
2. Review of dimensional reduction
    - CDT
    - Horava-Lifshitz gravity
    - Asymptotic safety
    - LQG
    - String theory (and other hints)
3. Motivations for challenging dimensional reduction (superluminality)
4. What is dimensional reduction really telling us?
5. Conclusions

Quantum gravity

- Standard model couplings are dimensionless - Gravitational coupling is *DIMENSIONFUL!* (In d-dimensions $[G_N] = 2 - d$)

Perturbative expansion

Perturbative QFT treatment of gravity in d-dimensions with loop order L scale with momentum p as (S. Weinberg, (1979).)

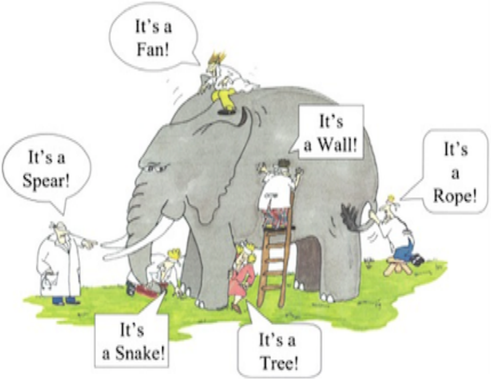
$$\int p^{A-[G_N]L} dp \tag{1}$$

Divergent for $[G_N] < 0$!

- Therefore, although gravity can be successfully formulated as an EFT at low energies ($J.Donoghue, arXiv/9512024v1.$), uncontrollable divergences appear in the high-energy perturbative expansion $Goroff + Sagnotti'86$
- Gravity as a perturbative QFT is *NOT* renormalizable!

Many approaches; few similarities

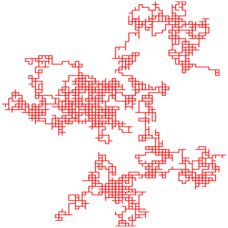
- Lack of experimental data at Planck scales $\rightarrow$ surplus of approaches to QG

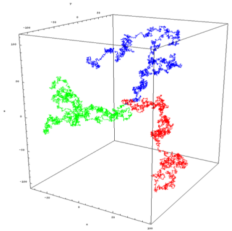


- Perhaps it is wise to look for similarities, e.g. thermodynamic nature of black holes... $\boxed{+\text{ dimensional reduction}}$

Dimensional reduction in CDT

- The spectral dimension D_S defines the effective dimension of a fractal geometry via a diffusion process
- D_S is related to the probability of return, $P_r(\sigma)$, for a random walk over an ensemble of triangulations after σ diffusion steps





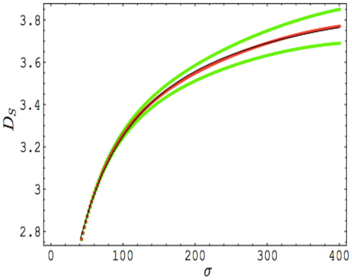
- The probability of return to the origin in asymptotically flat space is given by

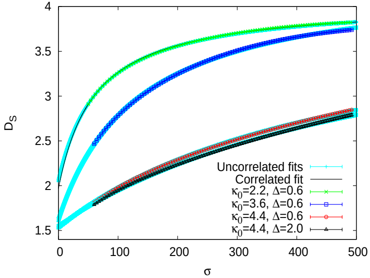
$$P_r(\sigma) = \frac{1}{\sigma^{d/2}}. \quad (2)$$

- Extract the spectral dimension D_S by taking the logarithmic derivative with respect to the diffusion time, giving

$$D_S = -2\frac{d\log\langle \mathrm{P_r}(\sigma)\rangle}{d\log\sigma}. \quad (3)$$

Dimensional reduction in CDT





$D_S(IR) = 4.02 \pm 0.10$,

$\boxed{D_S(UV) = 1.80 \pm 0.25}$

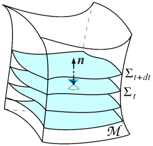
$D_S(IR) = 4.05 \pm 0.17$,

$\boxed{D_S(UV) = 1.970 \pm 0.27}$

$$D_S(\sigma) = a - \frac{b}{c+\sigma}$$

Dimensional reduction in Horava-Lifshitz gravity

- Anisotropic scaling of space and time in UV - Lorentz violating by construction
- Seems to have some connection to CDT... phase diagram arXiv:1002.3298 and spectral dimension arXiv:0902.3657



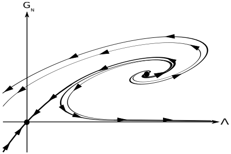
- In $D+1$-dimensional spacetime with anisotropic scaling exponent z spectral dimension is found to be

$$D_S = 1 + \frac{D}{z}. \tag{4}$$

- $z = 1$ in IR, $z = 3$ in UV.
- $\boxed{D_S(IR) = 4,\ D_S(UV) = 2.}$

Dimensional reduction in asymptotic safety

- Gravity is pertubatively nonrenormalizable - what about nonperturbatively?
- Growing body of evidence for UV fixed point



- At fixed point scaling of the effective graviton propagator dynamically reduces: arXiv:0108040v2

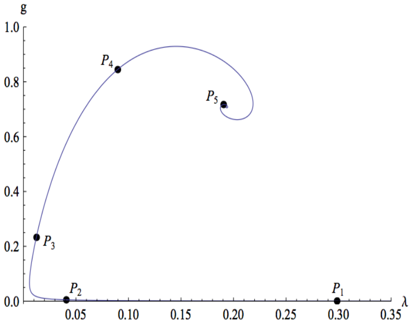
1. Classical regime (IR)

$$G(x,y) \propto \frac{1}{|x-y|^{d-2}} \tag{5}$$

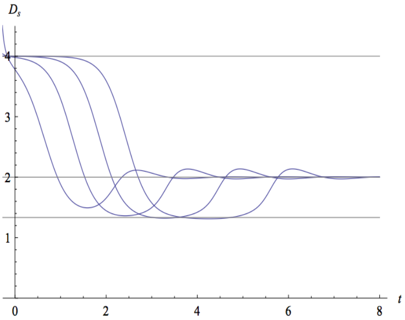
2. UV fixed point regime (UV)

$$G(x,y) \propto ln(\mu|x-y|) \tag{6}$$

Dimensional reduction in asymptotic safety



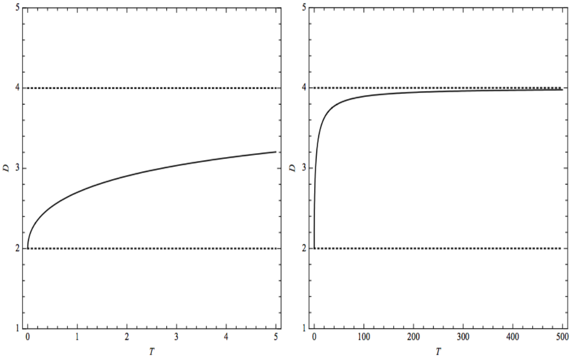
courtesy of Reuter + Saueressig
arXiv:1110.5224v1



$D_S(IR) = 4$,
$\boxed{D_S(UV) = 2}$

Dimensional reduction in loop quantum gravity

- Area spectrum $A \sim l_j^2$ for large areas, but $A \sim l_p l_j$ for small areas
- Modesto (*L.Modesto, arXiv/0812.2214.*) calculated spectral dimension in LQG by analysing area spectrum at varying length scales



- $\boxed{D_S(IR) = 4,\ D_S(UV) = 2.}$

Dimensional reduction in string theory (and other hints)

String theory

- High temperature gas of strings has free energy that behaves similar to a $2D$ QFT $\frac{F}{VT} \sim T$
- Spectral dimension in string theory implies dimensional reduction ($G.Calcagni, arXiv/1310.4957v2.$)

other hints

- Causal sets - Myrheim-Meyer dimension for a random causal set is ~ 2.38 (although D_S may actually increase...)
- Strong-coupling limit ($l_p \to \infty$) of Wheeler DeWitt equation implies $2-$dimensional behaviour ($Carlip$)

- If dimensional reduction is real we must accept some rather radical consquences:
  1. Relativistic symmetries are at the very least deformed
  2. Possible to break Lorentz invariance
  3. Gravitational waves cannot propagate
  4. Superluminal motion is possible

Superluminality

- As reported in $Amelino - Camelia, Phys.Rev., D87(12):123532, 2013$ most approaches describing dimensional reduction of the spectral dimension imply dispersion relations,

$$\frac{E}{p} = c_m = \sqrt{1 + (\lambda p)^{2\gamma}}. \tag{7}$$

- In $T.Sotiriou, phys.Rev., D84:104018(2011)$ an expression relating the spectal dimension to the ratio of the phase and group velocity, independent of any particular approach to QG, was derived.

$$D_S = 1 + d\frac{v_{phase}}{v_{group}} + ... \tag{8}$$

- For E.M waves in a vacuum $c_m = v_{group}/v_{phase} = 1$
- Clearly, for any degree of dimensional reduction $D_S < 4$ we have $c_m > 1$
- $\boxed{\text{Dimensional reduction implies superluminality}}$

Superluminality in CDT

- The canonical point in the de Sitter phase of CDT has a scale dependent spectral dimension of the form (supported by analytical multigraph approaches),

$$D_S = a - \frac{b}{c + \sigma}. \tag{9}$$

- CDT simulations yield a fit with $a = 4.06$, $b = 135$ and $c = 67$. *Coumbe + Jurkiewicz, JHEP* 1503(2015)151
- Integrating D_S gives a return probability

$$P_r = \frac{1}{\sigma^{a/2}\left(1 + \frac{c}{\sigma}\right)^{\frac{b}{2c}}}. \tag{10}$$

- as found in *Coumbe + Jurkiewicz, JHEP* 1503(2015)151 and *AJL, Phys.Rev.Lett.* 95.171301(2005) $a \approx 4$ and $\frac{b}{2c} \approx 1$. So to a good approximation one finds

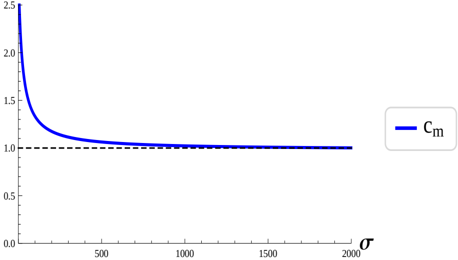
$$P_r \approx \frac{1}{\sigma^2 + c\sigma}. \tag{11}$$

superluminality in CDT

- Substituting the functional form for D_S into the expression $D_S = 1 + d\frac{v_{phase}}{v_{group}}$ gives a modified speed of light c_m of

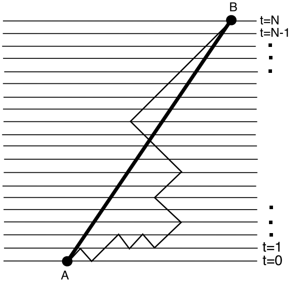
$$c_m = \frac{v_{group}}{v_{phase}} = \frac{d}{a - \frac{b}{c+\sigma}} - 1. \tag{12}$$

- Using the fit parameters determined from CDT calculations $a = 4.06$, $b = 135$ and $c = 67$ we can plot the modified speed of light c_m predicted by dimensional reduction in CDT



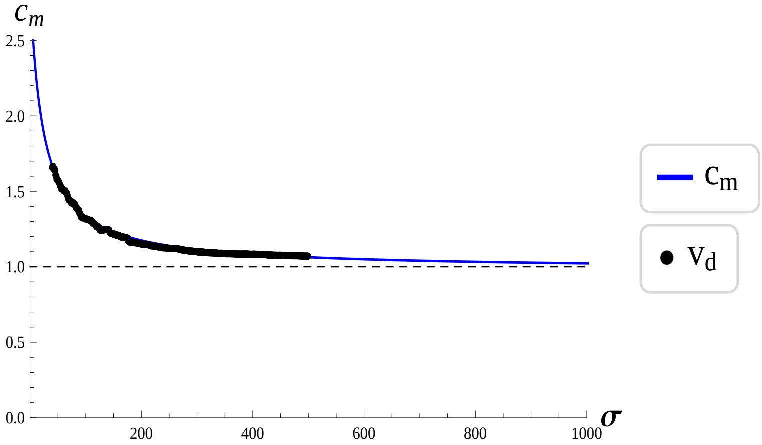
Numerical evidence for superluminality in CDT

- Test particle hops between adjacent simplices
- Unique label for combinatorial triangulations → can track the diffusion process
- Information about how path length varies with distance scale → define an effective velocity
- CDT defines an ensemble of triangulations with space-like hypersurfaces separated by time intervals t_N



$$\boxed{v_d = \frac{C\sigma}{t_d}}$$

Numerical evidence for superluminality in CDT



- Averaged over 1000 independent diffusions at canonical point. $C = 0.18$ such that extrapolates to $c_m = 1$ as $\sigma \to \infty$

An alternative derivation of superluminality in CDT (Mielczarek) arXiv:1503.08794v1

- Assumes fixed $D_S(IR) = 4$ and parameterises $D_S(UV) = 2 + \epsilon$, giving

$$D_S = 4 - \frac{2 - \epsilon}{1 + \sigma/c} \tag{13}$$

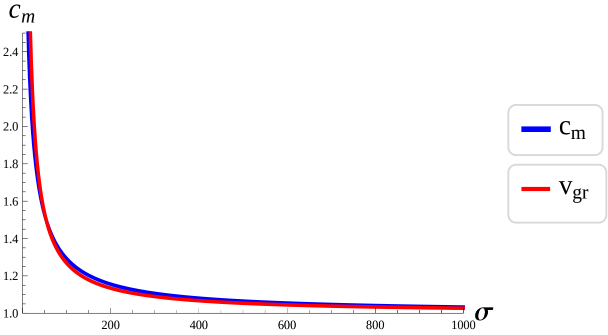
- Converts return probability into momentum via inverse Laplace transform obtaining approximate dispersion relations

$$\Omega_{IR}(p) \approx p + \frac{E_*}{15}(2 - \epsilon)\left(\frac{p}{E_*}\right)^3, \Omega_{UV}(p) \approx \frac{2}{3}E_* \frac{p}{E_*}^{3-3\epsilon} \tag{14}$$

$$\boxed{v_{gr} = 1 + \tfrac{3}{15}(2 - \epsilon)\left(\tfrac{E}{E_*}\right)^2}$$

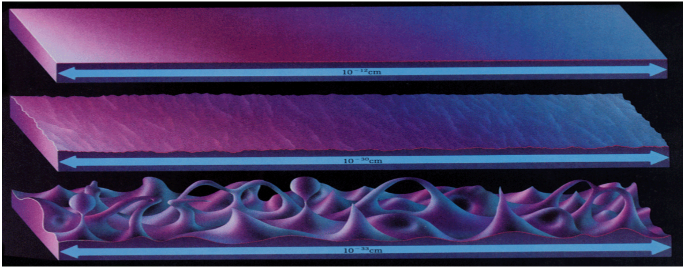
An alternative derivation of superluminality in CDT (Mielczarek) arXiv:1503.08794v1

- The fit parameter c in the fit function $a - b/(c + \sigma)$ can be related to the energy scale of dimensional reduction via $E_* = \frac{1}{\sqrt{c}}$
- If we assume that energy scale $E = 1/\Delta x$ then we can make a direct comparison between c_m and the independently derived v_{gr}



Lorentz invariance and astronomical observations

- If $c(E, \Delta x) > 1$ can we observe it?
- Fermi GBM/LAT collaboration using the Fermi Gamma-ray space telescope has severly constrained (linear) LIV even $E > E_P$ $(99\% CL)$



- How do we reconcile superluminality implied by dimensional reduction with Lorentz invariance and the empirical data?

Quantum gravity is missing something...

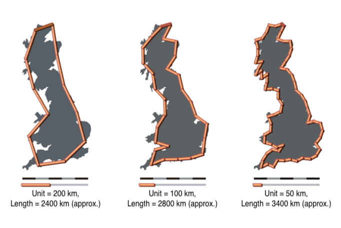
Fractal paths

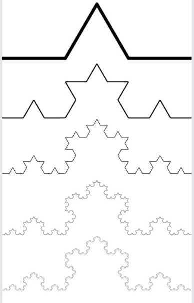
According to Feynman and Hibbs...

"*quantum mechanical paths are zig-zag lines, which are no-where differentiable, exhibiting self-similarity when viewed at different length scales*"

Abbott and Wise have shown that...

Quantum mechanical paths are fractals whose length depends on the measurement resolution





Do diffusing particles in CDT behave in a similar way?

Path length

- If diffusion paths in CDT behave similar to fractal QM paths, they must be scale dependent, parameterise by $\Gamma(\sigma)$
- What function $\Gamma(\sigma)$ rescales the path length σ when no DR ($P_r = \sigma^{-2}$) to give DR found in CDT ($\frac{1}{\sigma^2+c\sigma}$)?

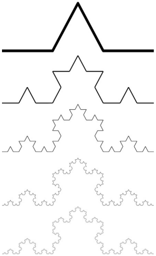
$$\frac{1}{\Gamma^2\sigma^2} = \frac{1}{\sigma^2 + c\sigma}, \tag{15}$$

giving

$$\Gamma = \sqrt{1 + \frac{c}{\sigma}} \tag{16}$$

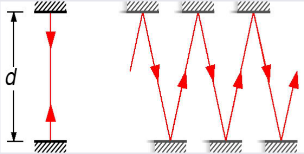
- Scale dependent path lengths explain why $c_m > c$

$$speed = \frac{PathLength}{time} = \frac{\Gamma \Delta l}{\Delta t} \tag{17}$$

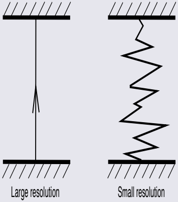


Time dilation

How do we preserve a constant speed of light? Simple: we adjust time.



$$\Delta t' = \gamma \Delta t$$



$$\Delta t' = \Gamma \Delta t$$

$\Gamma = \sqrt{1 + \frac{c}{\sigma}}$. As suggested by AJL $c = AG_N = AL_p^2$. Since $\sigma = \Delta x^2$ we get,

$$\Gamma = \sqrt{1 + \frac{AL_P^2}{\Delta x^2}} \tag{18}$$

Time dilates as a function of relative scale according to Γ

A dual description?

- Hausdorff introduced a new definition of length $\langle L\rangle$ that is independent of the measurement resolution Δx via a rescaling by the number of spatial Hausdorff dimensions d_H

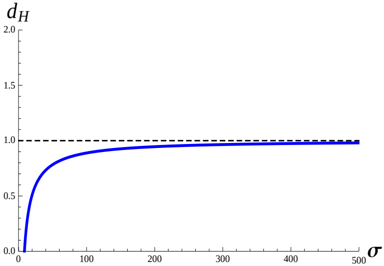
$$\langle L\rangle = \langle l\rangle \left(\Delta x\right)^{d_H - 1}. \tag{19}$$

- Ratio of invariant Hausdorff length $\langle L\rangle$ and variable length $\langle l\rangle$ is $1/\Gamma$, so that

$$\frac{\langle L\rangle}{\langle l\rangle} = \frac{1}{\Gamma} = \left(\Delta x\right)^{d_H - 1}. \tag{20}$$

- Giving the Hausdorff dimension of CDT diffusion paths

$$d_H = \frac{\ln\left(1/\Gamma\right)}{\ln\left(\Delta x\right)} + 1. \tag{21}$$



Temporal Hausdorff dimension

- Most DR of spectral dimension can be derived from dispersion relations of the type

$$E^2 = p^2 \left(1 + (\lambda p)^{2\gamma}\right). \tag{22}$$

- From this modified dispersion relation we obtain a modified speed of light c_m given by

$$c_m = \frac{E}{p} = \sqrt{1 + (\lambda p)^{2\gamma}}. \tag{23}$$

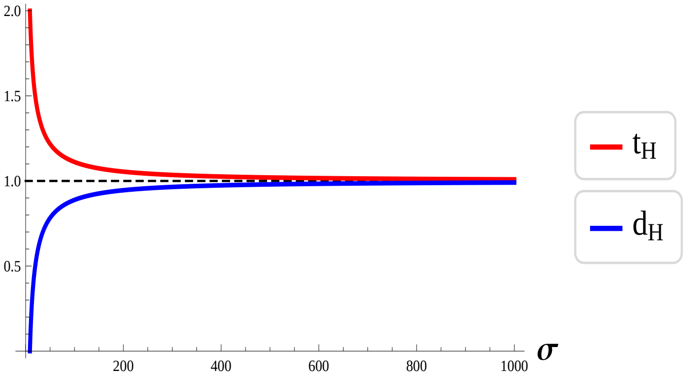
- In a spacetime with $(d_H + t_H)$ Hausdorff dimensions one finds the general form for the spectral dimension,

$$D_S = t_H + \frac{d_H}{1+\gamma}. \tag{24}$$

- Eq. (24) independent when $\gamma = 0$, and by using Eq. (21) we obtain

$$t_H = 1 - \frac{\ln(1/\Gamma)}{\ln(\Delta x)} \tag{25}$$

Scale dependent Wick rotation



- Spatial dimension transforming into a temporal dimension

$$\boxed{\text{Scale dependent Wick rotation?}}$$

Conclusions

1. Reviewed evidence for dimensional reduction
2. DR in CDT implies superluminality (2 independent derivations)
3. Numerical evidence for superluminality in CDT (data closely matches both predictions)
4. To maintain a constant speed of light
   time must dilate as a function of distance scale
5. Determined d_H and t_H for CDT diffusion paths — dual picture via scale dependent Wick rotation?

Thanks for listening!