Loop Quantum Cosmology in the Cosmic Microwave Background

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Quantum Gravity in Cracow 4
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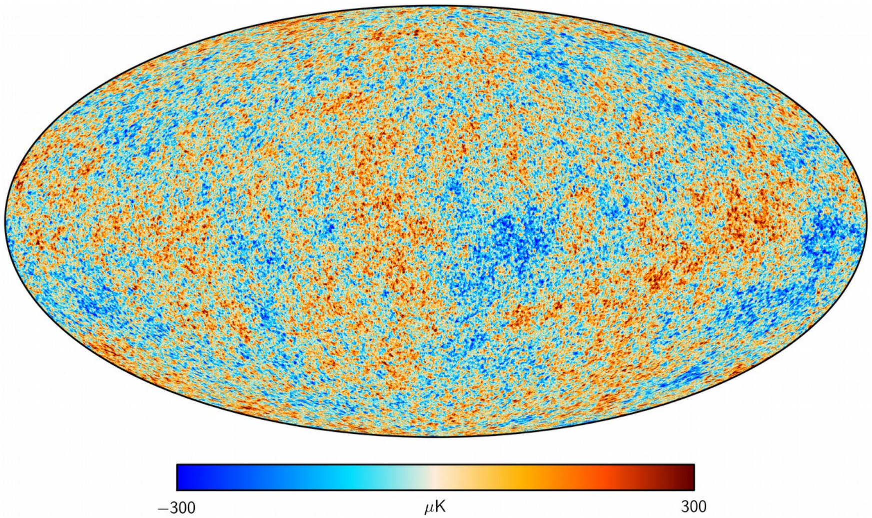
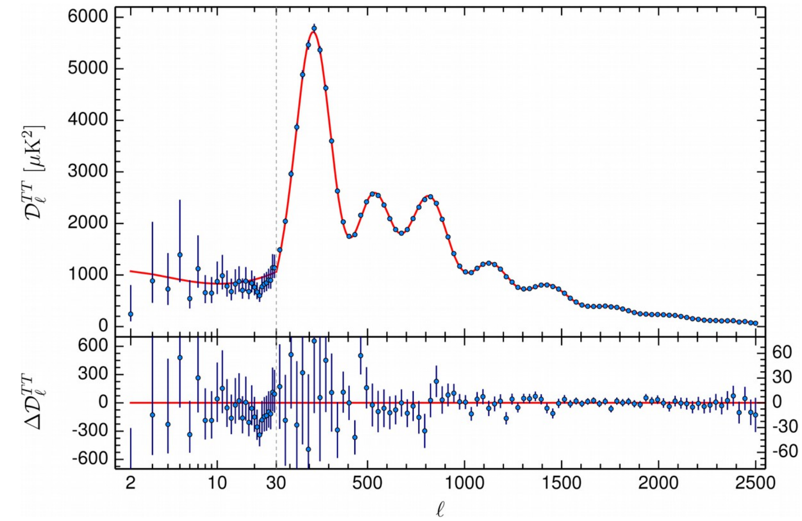
Motivations

General Relativity (GR)	100 y.a.
Cosmology as a Precision Science	80 y.a.
Cosmic Microwave Background (CMB) and *golden age* of GR	50 y.a.
Loop Quantum Gravity (LQG)	30 y.a.
Loop Quantum Cosmology (LQC)	20 y.a.
Planck and WMAP observations of the CMB	Now
Probing Quantum Gravity (QG)	Soon

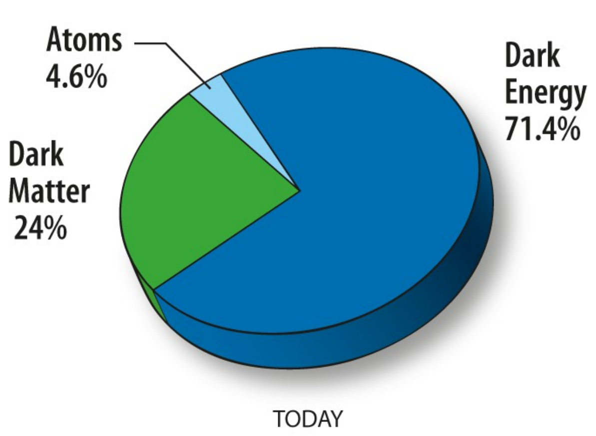
What are the observables likely to be experimental probes for QG?

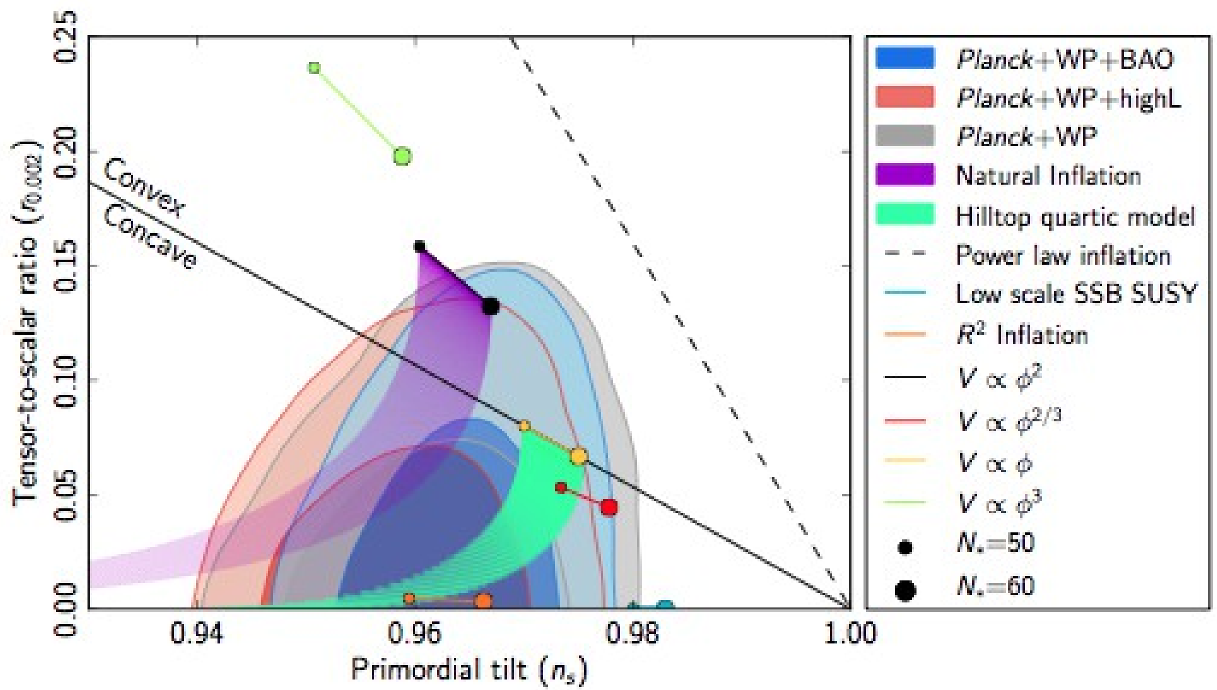
- Cosmic rays
- **CMB anisotropy power spectrum**
- CMB bispectrum





Normalized temperature inhomogeneities (left) and anisotropy power spectrum of the TT mode (right) of the CMB, Planck Collaboration (2015).





Prediction of the L-CDM model regarding the content of the universe (left) and constraints on different primordial inflation models (right), Planck Collaboration (2015).

Primordial power spectrum of the metric perturbation

$$\mathrm{g} = -a^2\left(1+\phi\right)\mathrm{d}\tau \otimes \mathrm{d}\tau + w_i \mathrm{d}\tau \otimes \mathrm{d}x^i + a^2\left(\delta_{ij} + h_{ij}\right)\mathrm{dx^i} \otimes \mathrm{dx^j}$$

perturbation

FLRW

$$\mathrm{g} \;=\; -a^2 \mathrm{d}\tau \otimes \mathrm{d}\tau + a^2\left(\delta_{ij} + h_{ij}\right)\mathrm{dx^i} \otimes \mathrm{dx^j}$$

in the **synchronous gauge**

$$\mathrm{h} = \begin{pmatrix} h & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 4\eta & 0 & 0 \\ 0 & -2\eta & 0 \\ 0 & 0 & -2\eta \end{pmatrix} + \begin{pmatrix} 0 & h_V^1 & 0 \\ h_V^1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & h_V^2 \\ 0 & 0 & 0 \\ h_V^2 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & h^+ & 0 \\ 0 & 0 & -h^+ \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & h^\times \\ 0 & -h^\times & 0 \end{pmatrix}$$

Definition of the primordial power spectrum,

$$\mathcal{P}_{\mathrm{T}}(k) = \frac{32Gk^3}{\pi}\left|\frac{v_k(\eta_{\mathrm{e}})}{a(\eta_{\mathrm{e}})}\right|^2$$

where **v=ah** and **η_e is the conformal time after a few e-fold of inflation.**

Experimental constraints on the amplitude and the scale dependence of P(k), for both scalar and tensor sectors.

Dynamics of the background in LQC+Massive S.F.

Energy density for a massive scalar field: $\rho = \rho_c \left(x^2 + y^2\right)$

Potential energy parameter, $x \equiv \frac{m\phi}{\sqrt{2\rho_c}}$ and **kinetic energy** parameter, $y \equiv \frac{\dot{\phi}}{\sqrt{2\rho_c}}$

The **modified Friedmann equation** and the **Klein-Gordon equation**...

$$H^2 = \frac{8\pi G\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

...are re-written as a system of **first order** differential equations,

$$\begin{cases} \dot{H} &= -8\pi G\rho_c y^2\left(1 - 2x^2 - 2y^2\right) \\ \dot{x} &= my \\ \dot{y} &= -3Hy - mx. \end{cases}$$

There are two time scales. Their ratio defines the dimensionless parameter

$$\Gamma \equiv \frac{m}{\sqrt{24\pi G\rho_c}}$$

When $\Gamma \equiv \frac{m}{\sqrt{24\pi G\rho_c}}$ is small,

the dynamics splits into three phases.

1) Contracting phase

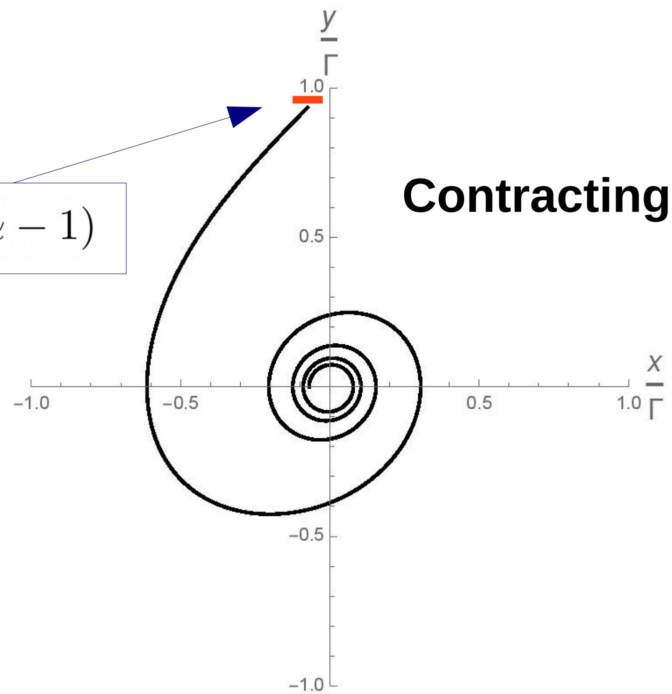
2) Bouncing phase

3) **Slow-roll inflation**

Dynamics of the background

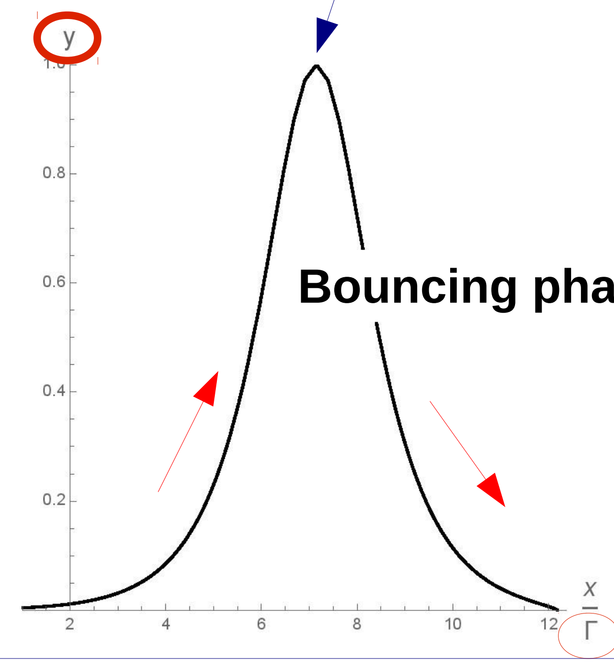
$$\sqrt{\frac{\rho(t)}{\rho_c}} = \frac{m}{\alpha\sqrt{24\pi G\rho_c}}\left\{1 - \frac{1}{2\alpha}\left[mt + \tfrac{1}{2}\sin(2mt + 2\theta_0)\right]\right\}^{-1}$$

$$t_A = \tfrac{2}{m}(\alpha - 1)$$



Contracting phase

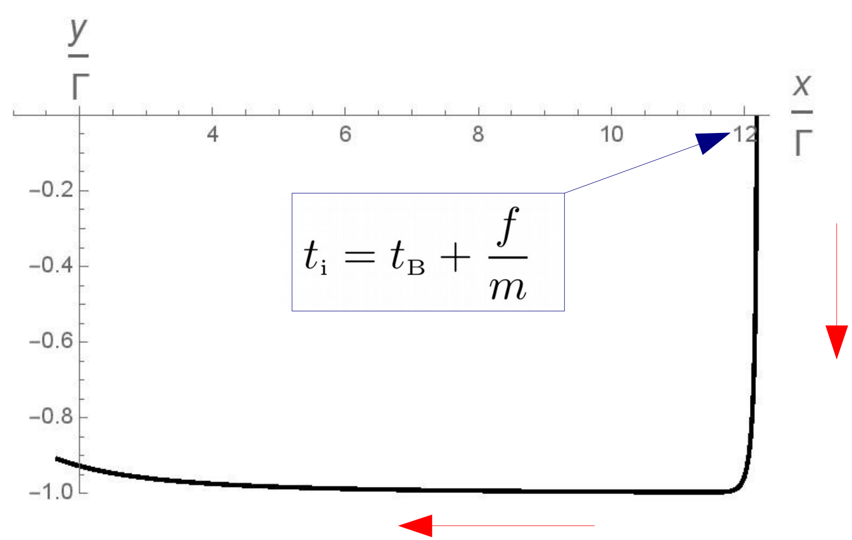
$$t_B = t_A + \frac{1}{m\,|\cos\theta_A|}$$



Bouncing phase

$$\begin{cases} y\,(t) = \{1 + 24\pi G\rho_c(t - t_B)^2\}^{-\frac{1}{2}} \\ x\,(t) = x_B + \varepsilon\frac{m}{\sqrt{24\pi G\rho_c}}\operatorname{arcsinh}\left(\sqrt{24\pi G\rho_c}\,(t - t_B)\right) \end{cases}$$

$$\begin{cases} y = -\varepsilon\frac{m}{\sqrt{24\pi G\rho_c}} \\ \dot{x} = my \end{cases}$$



$$t_i = t_B + \frac{f}{m}$$

Slow-roll inflation

Dynamics of the perturbations

The equation of motion for **tensor modes** is

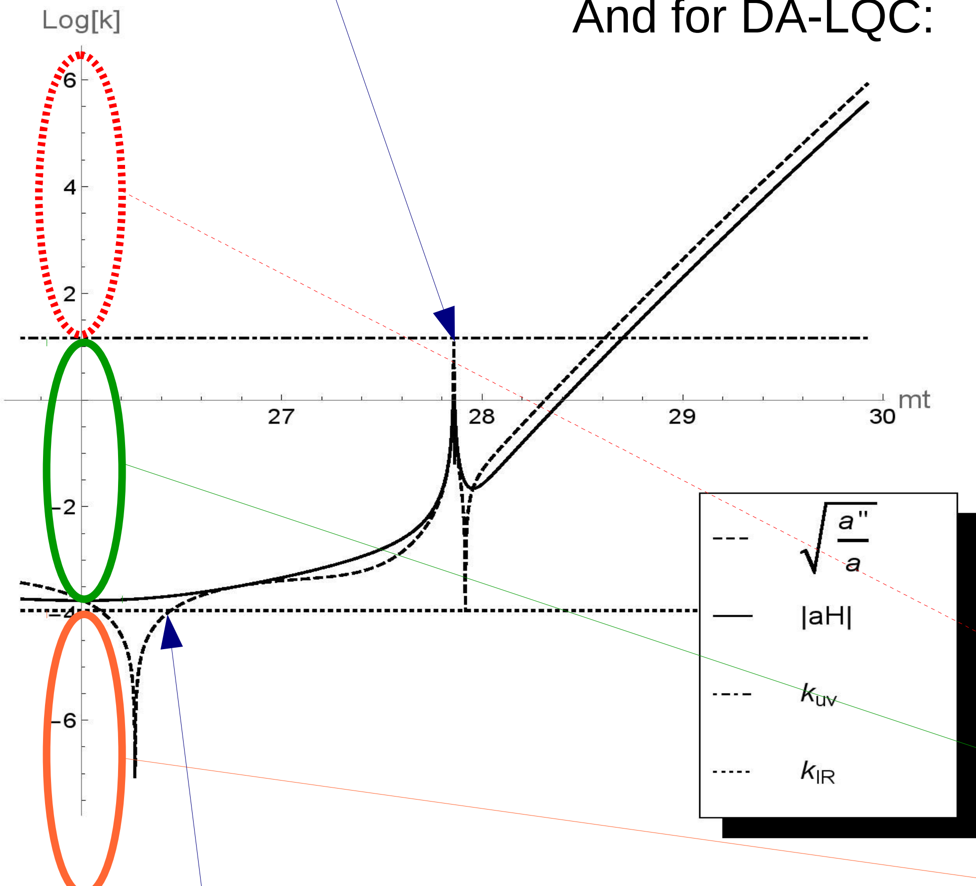
$$v_k''(\eta) + \left(k^2 - \frac{a''}{a}\right) v_k(\eta) = 0$$

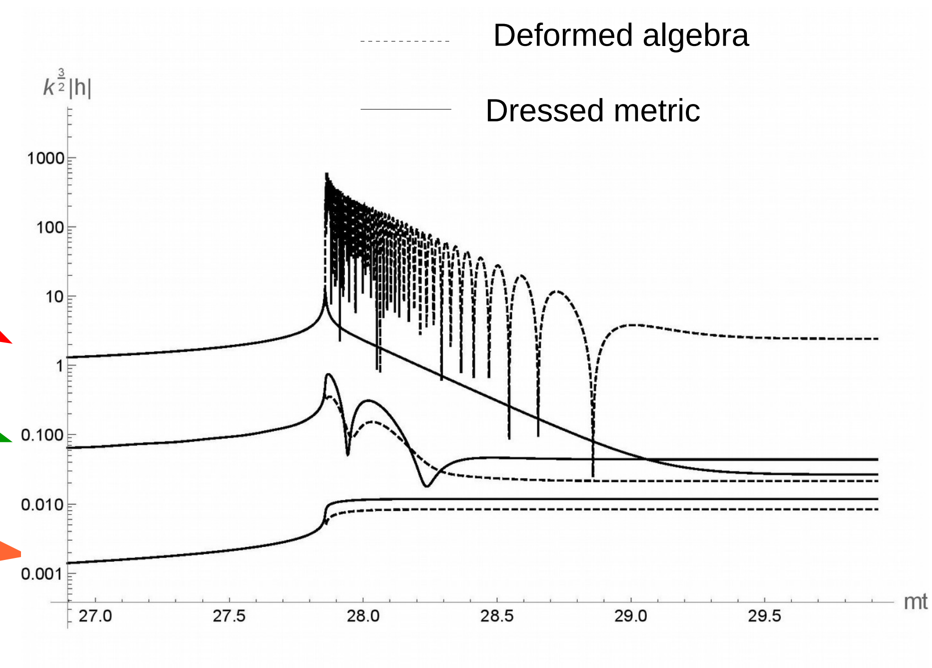
$$k_{\mathrm{UV}} = \sqrt{8\pi G\rho_c}$$

And for DA-LQC:

$$v_k''(\eta) + \left(\boldsymbol{\Omega} k^2 - \frac{z_{\mathrm{T}}''}{z_{\mathrm{T}}}\right) v_k(\eta) = 0$$

where $\boldsymbol{\Omega} \equiv 1 - 2\dfrac{\rho}{\rho_c}$ and $z_{\mathrm{T}} \equiv \dfrac{a}{\sqrt{\boldsymbol{\Omega}}}$





$$k_{\mathrm{IR}} = \frac{1}{3\sqrt{2}}\left(\frac{m^2\sqrt{24\pi G\rho_c}}{|\cos\theta_{\mathrm{A}}|}\right)^{1/3}$$

Calculation of the power spectrum

Definition of the power spectrum:

$$\mathcal{P}_{\mathrm{T}}(k) = \frac{32Gk^3}{\pi}\left|\frac{v_k(\eta_{\mathrm{e}})}{a(\eta_{\mathrm{e}})}\right|^2$$

IR limit

Solution to the equation of motion at large scales

$$v_{k\to 0}(\eta) = \alpha_k a(\eta) + \beta_k a(\eta)\int_{\eta_\star}^{\eta}\frac{d\eta'}{a^2(\eta')} + \mathcal{O}(k^2)$$

$$\mathcal{P}_{\mathrm{T}}(k)^{\mathrm{IR}} = \frac{32Gk^3}{\pi}\left|\alpha_k + \beta_k I(\eta_{\mathrm{e}})\right|^2$$

The coefficients α and β are calculated with a matching in the remote past, the integral is computed with the analytical expressions of the background variables.

$$\mathcal{P}_{\mathrm{T}}(k)^{\mathrm{IR}} = \frac{4G}{9\pi}m^2|1+\mathcal{I}+\mathcal{J}|^2$$

$$\mathcal{I} \equiv -\frac{1}{|\cos\theta_{\mathrm{A}}|}\ln\left(\frac{1}{2}\Gamma\sqrt{\frac{|\cos\theta_{\mathrm{A}}|}{f}}\right)$$

$$\mathcal{J} \equiv \frac{\sqrt{3}}{2}\frac{1}{|\cos\theta_{\mathrm{A}}|}\left|\frac{\Gamma}{x_{\mathrm{i}}}\right|$$

$$x_{\mathrm{i}} = x_{\mathrm{A}} - 2\varepsilon\frac{m}{\sqrt{24\pi G\rho_{\mathrm{c}}}}\ln\left(\frac{m}{2\sqrt{24\pi G\rho_{\mathrm{c}}}}\sqrt{\frac{|\cos\theta_{\mathrm{A}}|}{f}}\right)$$

UV limit, Dressed Metric

A well-know calculation leads to

$$\mathcal{P}_{\mathrm{T}}(k)^{\mathrm{UV}} = \frac{16G}{\pi}m^2\left|\frac{x_{\mathrm{i}}}{\Gamma}\right|^2$$

with a tilt,

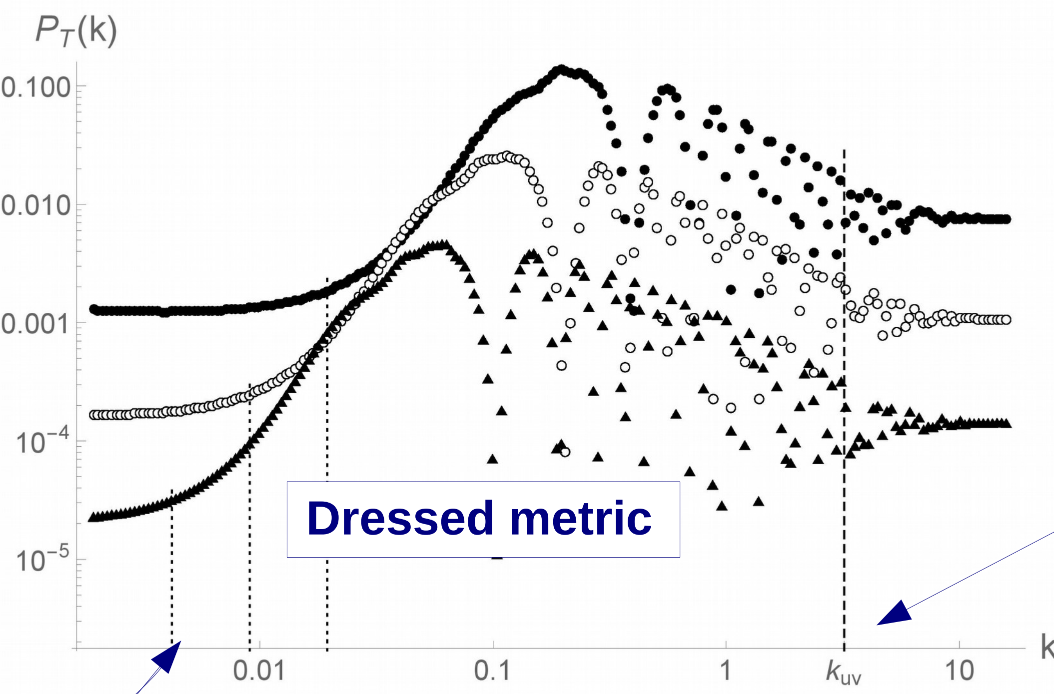
$$\frac{\mathrm{d}\ln\mathcal{P}_{\mathrm{T}}(k)^{\mathrm{UV}}}{\mathrm{d}\ln k} = -6\left|\frac{\Gamma}{x_{\mathrm{i}}}\right|^2$$

Deformed Algebra

$$v_{k\to\infty} \propto \exp\left(k\times\int_{\Delta\eta}\sqrt{|\mathbf{\Omega}|}\,d\eta\right)$$

Varying the mass

m=10$^{-(2+p/2)}$ with p=0,1,2



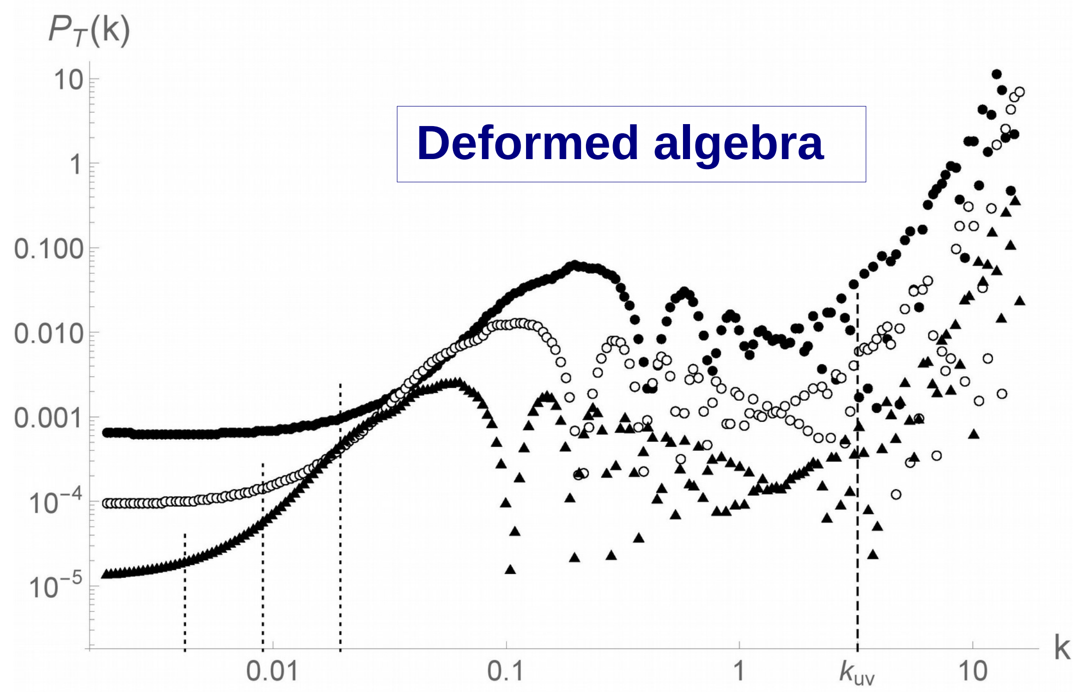
$$\mathcal{P}_{\mathrm{T}}(k)^{\mathrm{UV}} = \frac{16G}{\pi} m^2 \left|\frac{x_{\mathrm{i}}}{\Gamma}\right|^2$$

$$k_{\mathrm{UV}} = \sqrt{8\pi G \rho_c}$$

$$v_{k\to\infty} \propto \exp(k \times \int_{\Delta\eta} \sqrt{|\boldsymbol{\Omega}|}\, d\eta)$$

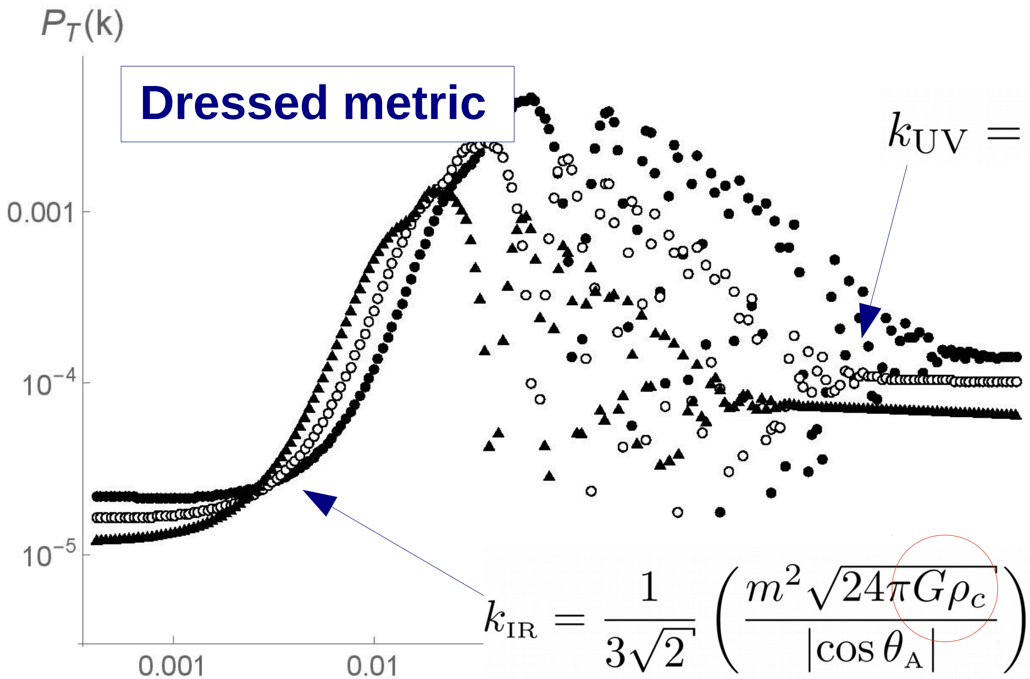
$$\mathcal{P}_{\mathrm{T}}(k)^{\mathrm{IR}} = \frac{4G}{9\pi} m^2 |1 + \mathcal{I} + \mathcal{J}|^2$$

$$k_{\mathrm{IR}} = \frac{1}{3\sqrt{2}} \left(\frac{m^2\sqrt{24\pi G\rho_c}}{|\cos\theta_{\mathrm{A}}|}\right)^{1/3}$$



Varying the critical density

$\rho_c = 0{,}41 \times 10^{-p}$ with $p = 0,1,2$



$$k_{\rm UV} = \sqrt{8\pi G \rho_c}$$

$$k_{\rm IR} = \frac{1}{3\sqrt{2}} \left(\frac{m^2 \sqrt{24\pi G \rho_c}}{|\cos\theta_{\rm A}|} \right)^{1/3}$$

$$\mathcal{P}_{\rm T}(k)^{\rm UV} = \frac{16G}{\pi} m^2 \left| \frac{x_{\rm i}}{\Gamma} \right|^2$$

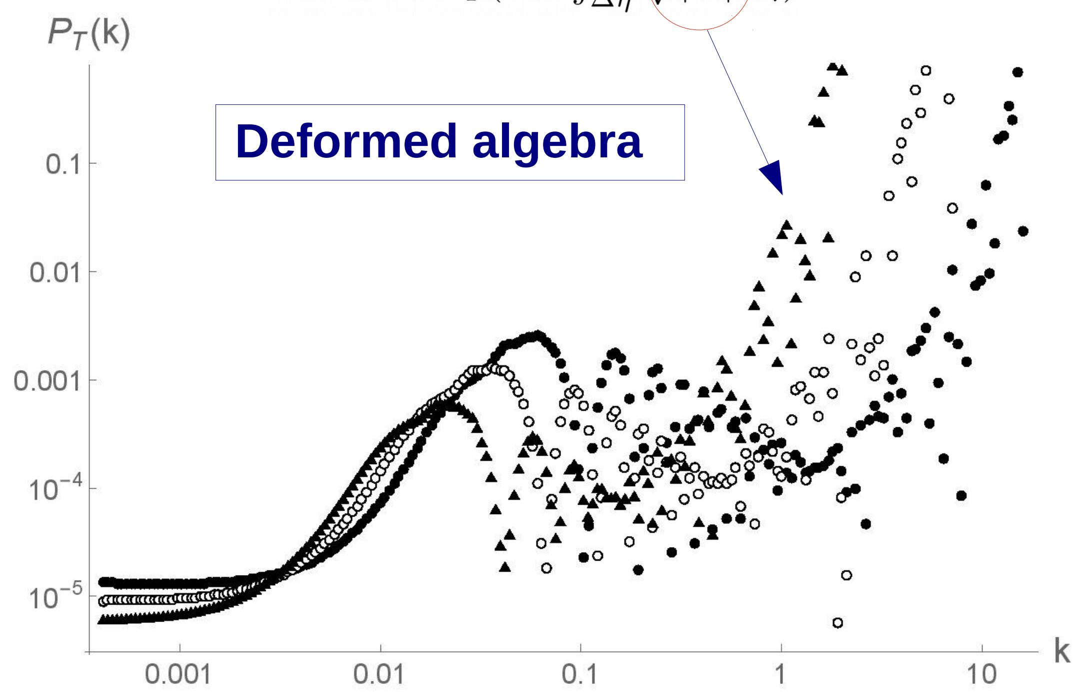
$$x_{\rm i} = x_{\rm A} - 2\varepsilon \frac{m}{\sqrt{24\pi G \rho_{\rm c}}} \ln \left(\frac{m}{2\sqrt{24\pi G \rho_{\rm c}}} \sqrt{\frac{|\cos\theta_{\rm A}|}{f}} \right)$$

$$v_{k\to\infty} \propto \exp(k \times \int_{\Delta\eta} \sqrt{|\mathbf{\Omega}|}\, d\eta)$$

$$\mathcal{P}_{\rm T}(k)^{\rm IR} = \frac{4G}{9\pi} m^2 |1 + \mathcal{I} + \mathcal{J}|^2$$

$$\mathcal{I} \equiv -\frac{1}{|\cos\theta_{\rm A}|} \ln \left(\frac{1}{2} \Gamma \sqrt{\frac{|\cos\theta_{\rm A}|}{f}} \right)$$

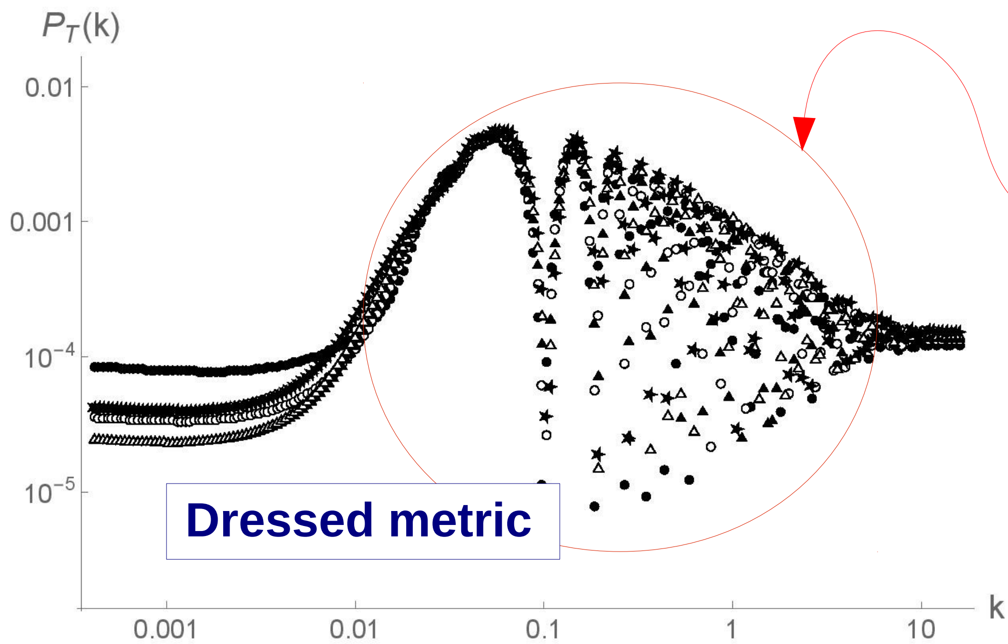
$$\mathcal{J} \equiv \frac{\sqrt{3}}{2} \frac{1}{|\cos\theta_{\rm A}|} \left| \frac{\Gamma}{x_{\rm i}} \right|$$

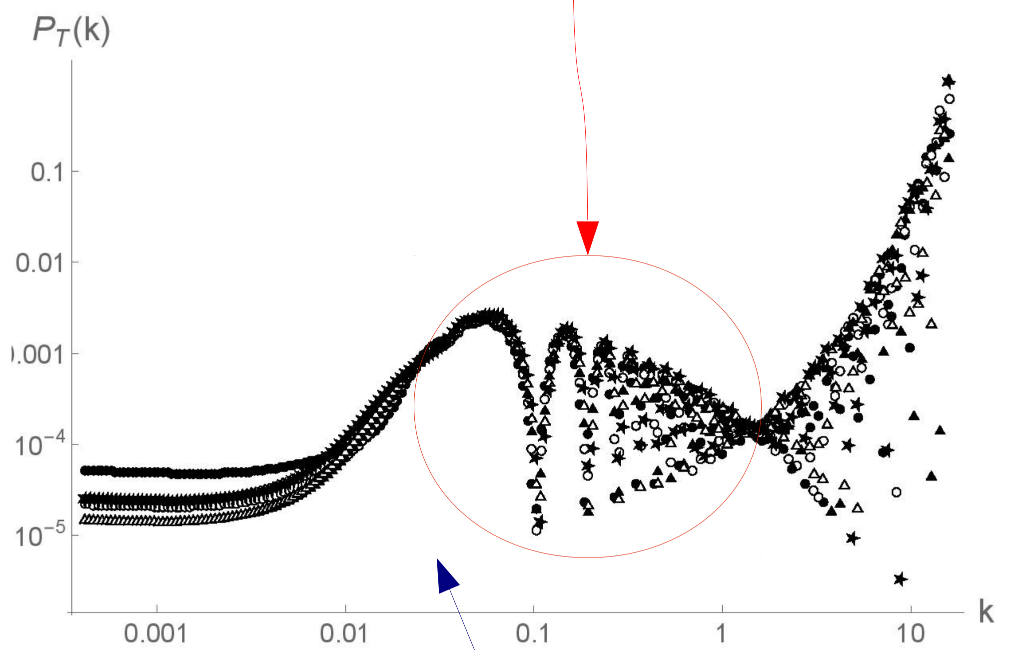


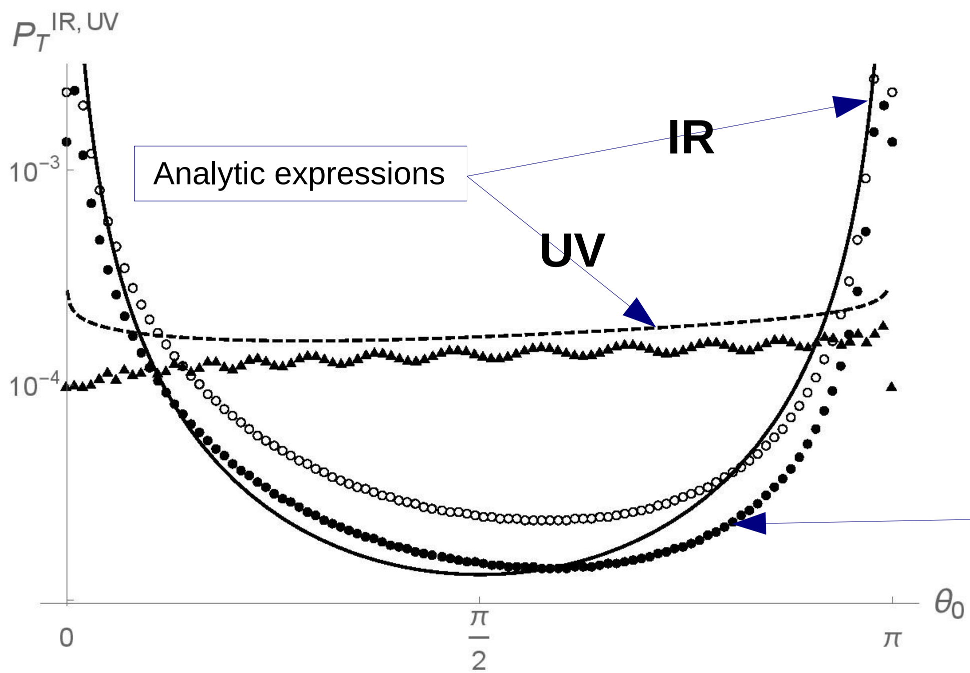
Varying the initial phase

$$\begin{cases} x(t) = \sqrt{\dfrac{\rho(t)}{\rho_c}} \sin(mt + \theta_0) \\ y(t) = \sqrt{\dfrac{\rho(t)}{\rho_c}} \cos(mt + \theta_0) \end{cases}$$

Oscillations in both approache
and no dependence over the phase.

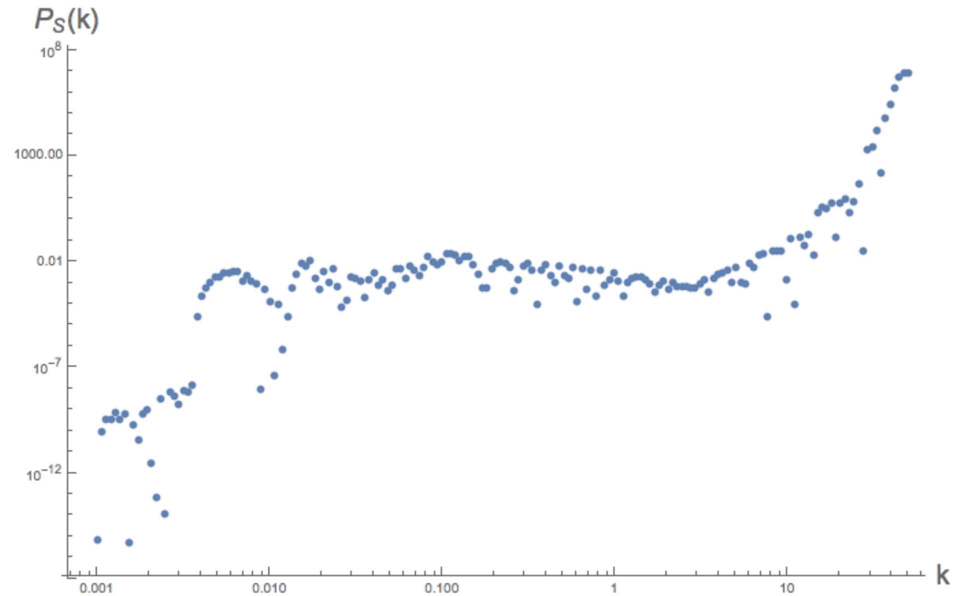






Deformed algebra

Current issues and perspectives

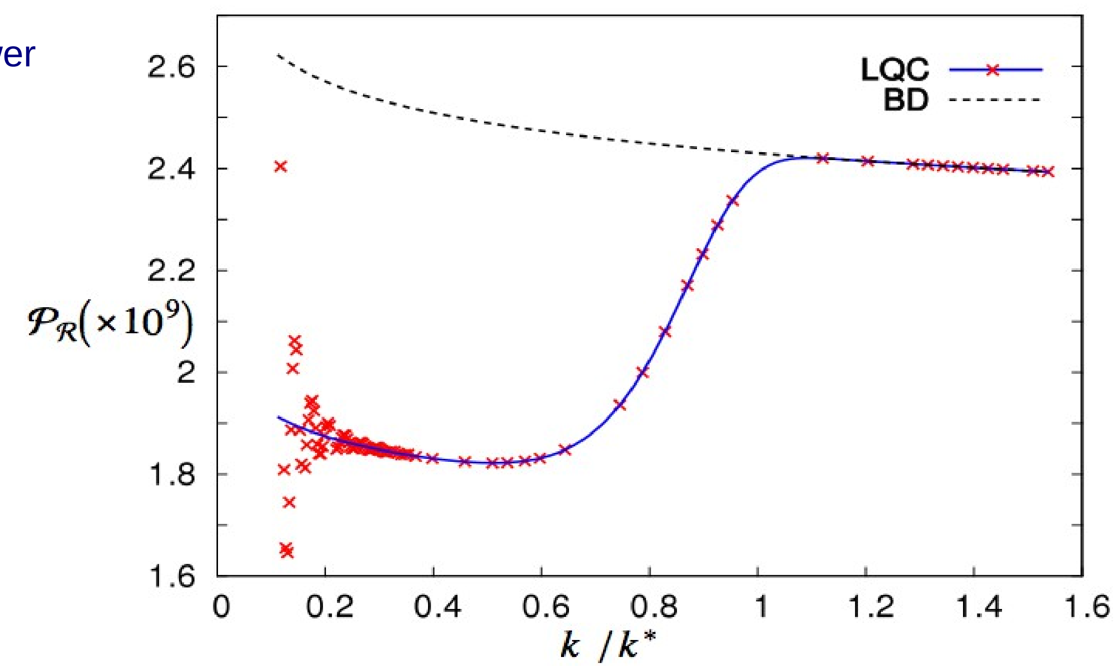


Preliminary results for the primordial scalar power spectrum in the deformed algebra approach.
Credit: Susanne Schander.

The same work for the **scalar perturbations** has to be done in the Deformed Algebra approach. Qualitatively, the same power spectrum as for the tensor modes is expected.

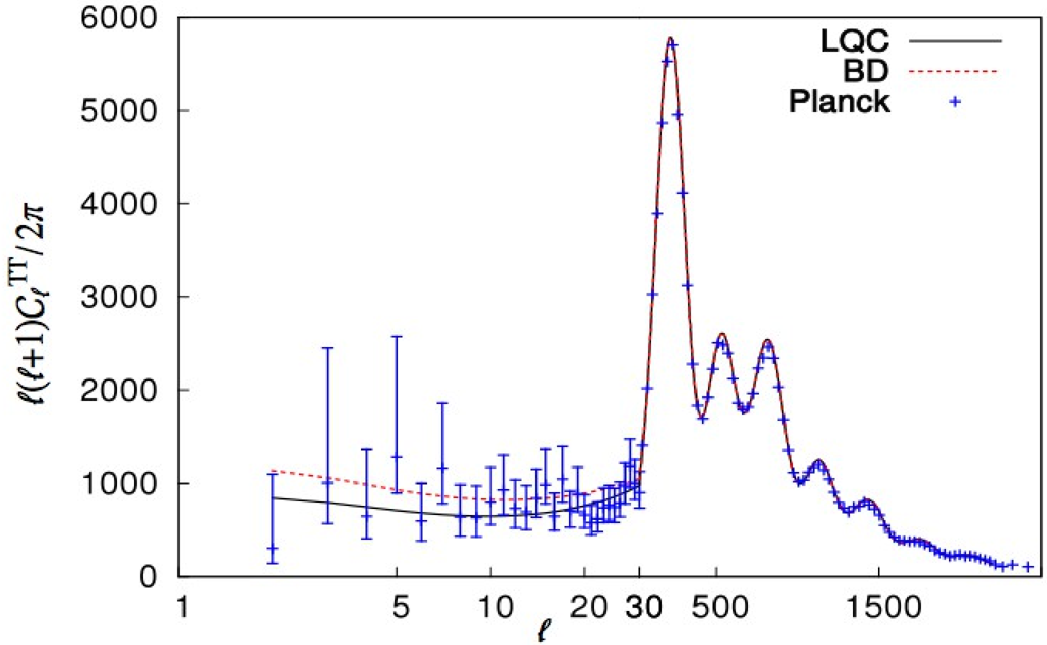
In the Dressed Metric approach, the scalar spetrum has already been obtained numerically, however with **initial conditions at the bounce** (Ashtekar & Agullo, 2011).

Needs the appropriate QFT vaccum at such high curvature.



Primordial scalar power spectrum in the dressed metric approach.
Credit: A. Ashtekar and B. Gupt (in preparation).

Current issues and perspectives



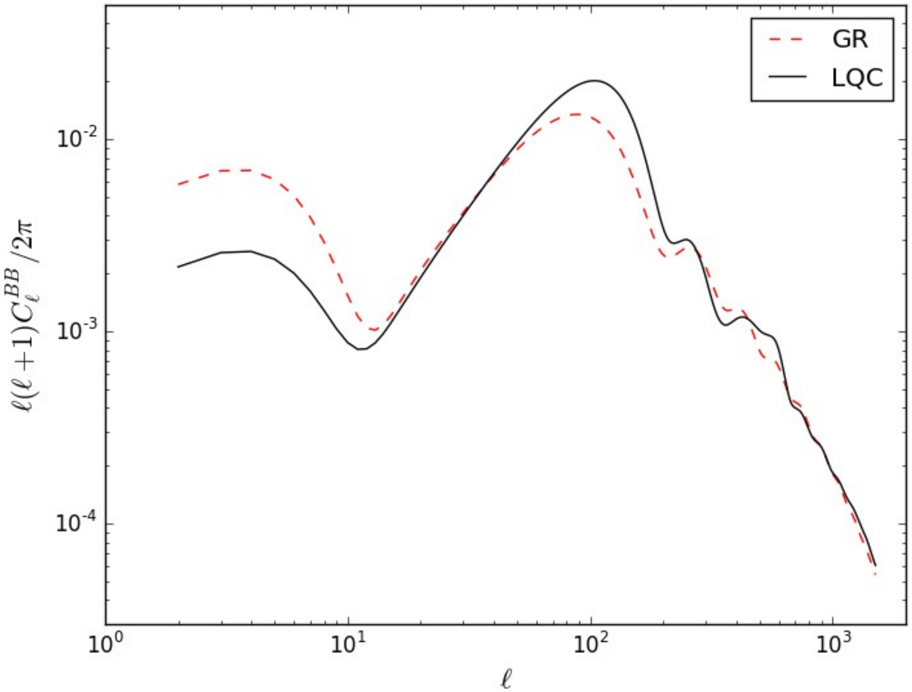
CMB-TT power spectrum, LQC vs. BD. Credit: A. Ashtekar and B. Gupt (in preparation).

The CMB-TT power spectrum can be reconstructed.

1) Constraints on the parametrs of the model (defined by the experimental uncertainties).

2) Or a better fit to the data.

The CMB-B modes have not been measured at sufficient precision for isolating the primordial gravitational wave component. However, forthcoming experiment will do.



CMB-BB power spectrum, LQC vs. BD.Credit: B.Bolliet.