Hartle-Hawking Wave function in Causal Sets

(Glaser,Surya arXiv:1410.8775)

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Outline

The questions I hope to answer in this talk:

- What is a causal set?
- Why do we use 2d orders as a model system?
- How do we define the Hartle-Hawking wave function for causal sets?
- And how did we implement this definition in our Monte-Carlo code?

Outline

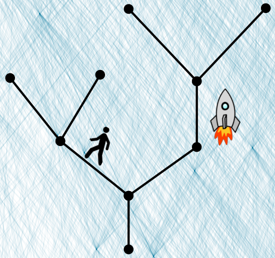
What is a causal set?

The 2-d orders

MC for 2d orders

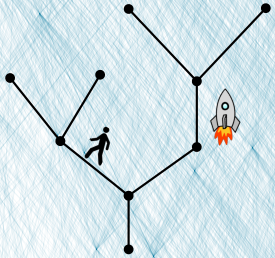
The HH wave function

Number + order = Geometry



- Each point is one event, with space-time volume V_d
- Each line is a time-like connection between two events
- This information characterises a discrete Poincaré invariant space-time

Number + order = Geometry



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Space-time as a partially ordered set

Number + order = Geometry

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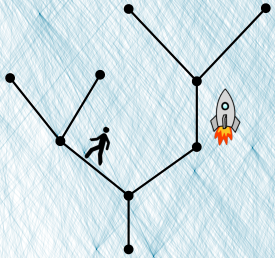
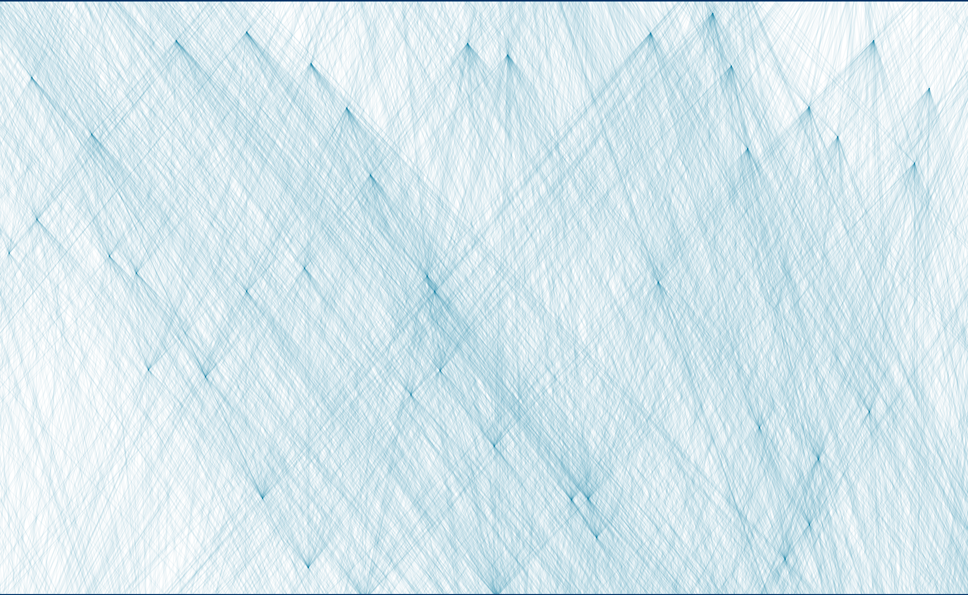


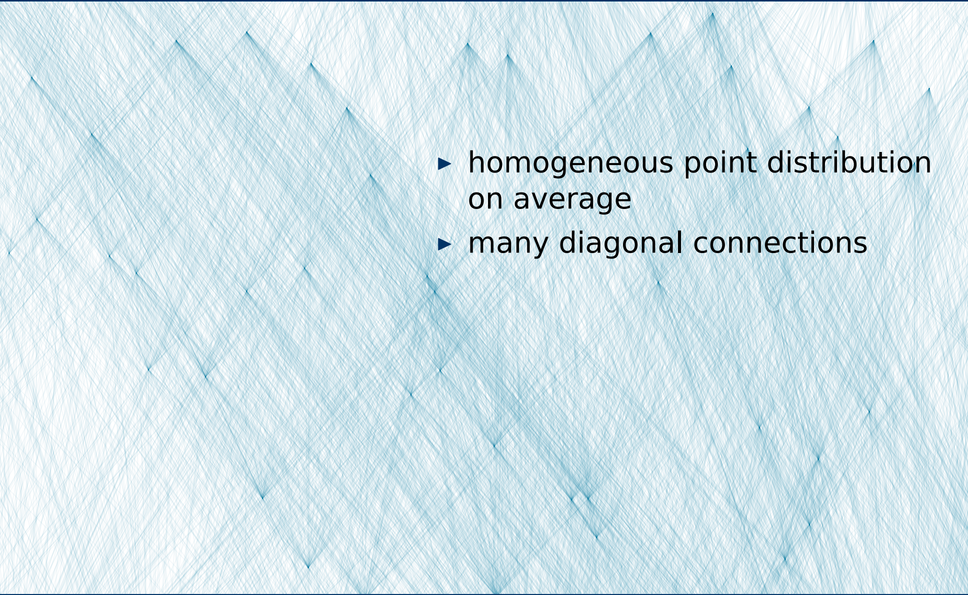
- **reflexive** for all $x \in \mathcal{C}$ $x \preceq x$
- **transitive** for all $x, y, z \in \mathcal{C}$ and $x \preceq y$ and $y \preceq z$ then $x \preceq z$
- **antisymmetric** if $x, y \in \mathcal{C}$ and $x \preceq y \preceq x$ then $x = y$
- **locally finite** for all $x, y \in \mathcal{C}$ $|I(x, y)| < \infty$

Space-time as a partially ordered set

A manifold-like causal set



A manifold-like causal set



- homogeneous point distribution on average
- many diagonal connections

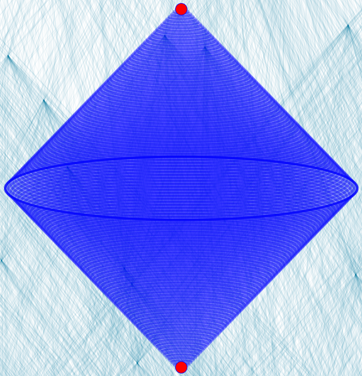
A manifold-like causal set

- pick N points from $\mathcal{M}$ according to a Poisson distribution

$$P(m, V, \rho) = \frac{(\rho V)^m}{m!} e^{-\rho V}$$

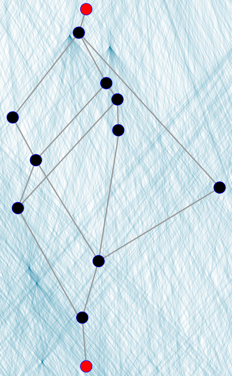
- partial order is induced through the causal structure of the manifold

Alexandrov intervals



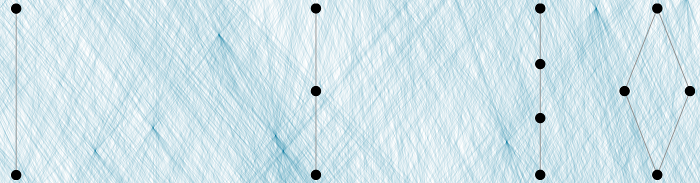
$$V_{0\,d}(x,y) = S_{d-2}\frac{1}{d(d-1)2^{d-1}}\tau_{x-y}^{d}$$

Alexandrov intervals



$$I[x,y] := \{z | x \prec\prec z \prec\prec y\}$$

Alexandrov intervals



link

1-interval

2-intervals

The action

Assuming the discreteness is at the Planck scale $l = l_p$

$$\frac{1}{\hbar} S_{2D} = N - 2N_0 + 4N_1 - 2N_2$$

Where

- N $=$ # of elements
- $N_0 =$ # of relations between elements
- $N_1 =$ # of 1-intervals
- $N_2 =$ # of 2-intervals

The action

Assuming the discreteness is at the Planck scale $l = l_p$

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This expression is motivated by the d'Alembertian operator and can be generalized to any dimension

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This expression is motivated by the d'Alembertian operator and can be generalized to any dimension

In the large density limit this action will fluctuate strongly

Non-locality scale

Sorkins solution

Introduce a smearing function $\epsilon = \left(\frac{l_p}{l}\right)^d$

$$f_2(n,\epsilon) := (1-\epsilon)^n \left(1 - \frac{2\epsilon n}{(1-\epsilon)} + \frac{\epsilon^2 n(n-1)}{2(1-\epsilon)^2}\right)$$

(Sorkin arXiv:gr-qc/0703099)

Non-locality scale

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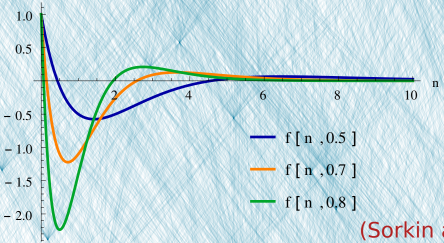

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$$\frac{1}{\hbar} S_{2D}(\epsilon) = 4\epsilon \left(N - 2\epsilon \sum_{n=0}^{N-2} N_n f_2(n,\epsilon)\right)$$

N_n is the number of n-intervals

(Sorkin arXiv:gr-qc/0703099)

Outline

What is a causal set?

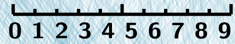
The 2-d orders

MC for 2d orders

The HH wave function

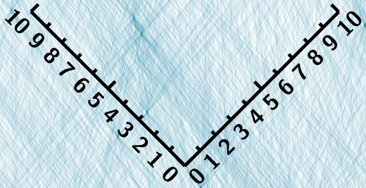
A simpler class of causal sets

The numbers from 0 to 10 define a total order



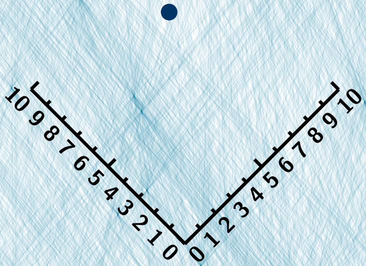
A simpler class of causal sets

We can take two such total order to define a coordinate grid



A simpler class of causal sets

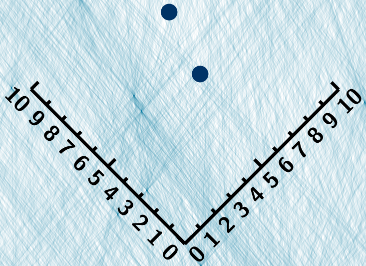
A pair of two numbers defines an element.



$(8, 7)$

A simpler class of causal sets

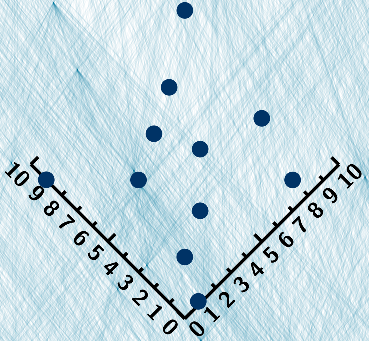
Randomly pair the numbers to define a 2d order



$(5,6),(8,7)$

A simpler class of causal sets

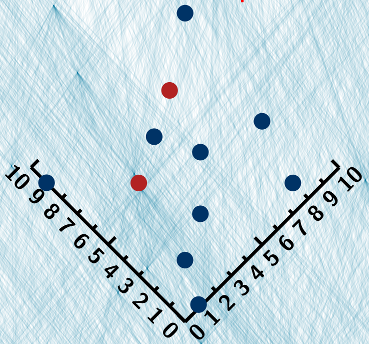
Randomly pair the numbers to define a 2d order



$$(0,1),(1,8),(2,2),(3,4),(4,9),(5,6),(6,3),(7,5),(8,7),(9,0),(10,10)$$

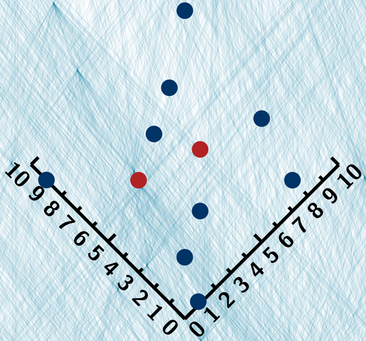
A simpler class of causal sets

Total order on numbers defines a partial order on elements



$(6,3) \prec (8,7)$ because $6 < 8$ and $3 < 7$

Total order on numbers defines a partial order on elements



$(6,3)$ not related to $(5,6)$ because $6>5$ and $3<6$

(Dis)Advantages

In favour of 2d orders

- simple encoding on the computer, with ergodic MC move
- inbuildt embedding makes visualisation easy
- the ‘typical’ 2d order is a sprinkling into flat space
- in a sense fixed to 2d, makes clear which action

Against 2d orders

- very simple, will only capture part of dynamics
- can not capture entropy effects
- is 2d gravity at best

Outline

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Monte-Carlo Simulations of arbitrary 2d orders

The action can be used for Monte Carlo Simulations

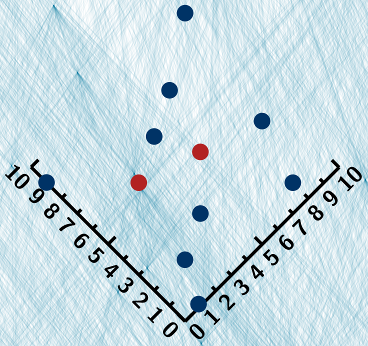
$$Z_N = \sum_{C \in \Omega_{2d}} e^{-\frac{\beta}{\hbar} S_{2D}(C,\epsilon)}$$

β is a Wick rotated inverse temperature and Ω_{2d} is the class of 2d orders

(Surya arXiv:1110.6244)

The MC move on these orders

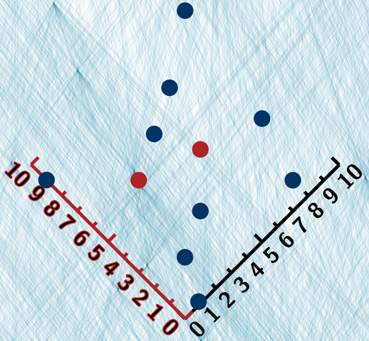
Pick 2 points.



$(6, 3)$ and $(5, 6)$

The MC move on these orders

Randomly decide on one direction



$(\color{red}{6}, 3)$ and $(\color{red}{5}, 6)$

The MC move on these orders

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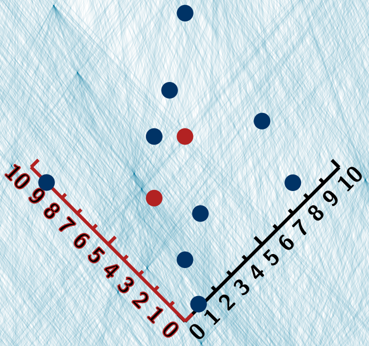
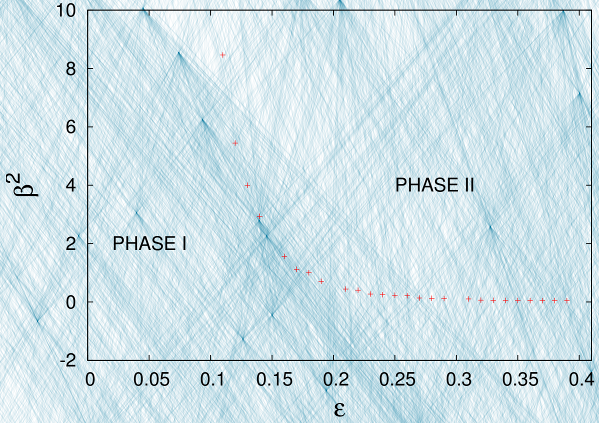
Switch their coordinates in that direction



$(5, 3)$ and $(6, 6)$

Phase transition



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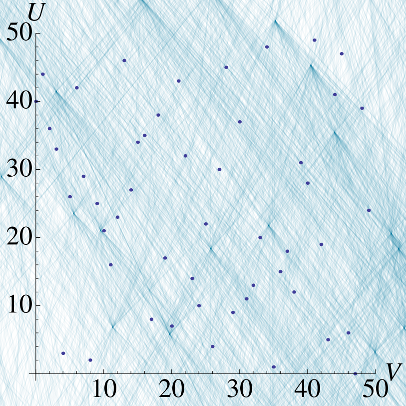
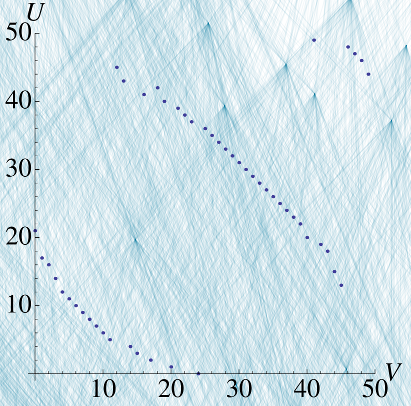


Phase I

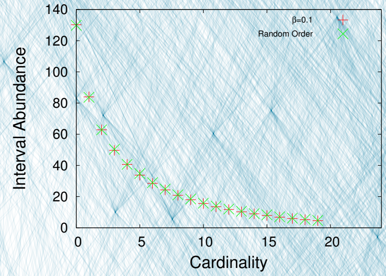


Phase II

(Surya arXiv:1110.6244)

A sign of manifold-likeness?

Counting the number of sub intervals of a given size



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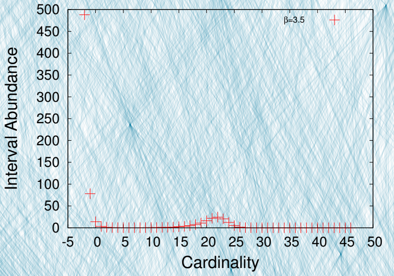

Phase II

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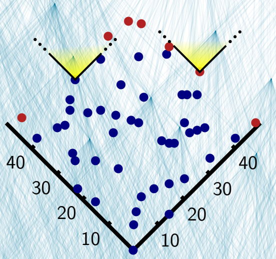
Hartle Hawking wave function

Continuum

$$\Psi_0(h_{ab}, \Sigma) = A \sum_M \int dg^E e^{-I_E(g)}$$

- Integrate over euclidean geometries
- **zero boundary condition** to final geometry (h_{ab}, Σ)

Hartle Hawking wave function



Causal Set

$$\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{C \in \Omega_N} e^{-\frac{1}{\hbar}\beta S(C)}$$

- Sum over all Causal Sets
- **single initial element** to $\mathcal{N}_f$ antichain
- in CDT one might call this the loop-loop correlator $G(0, \mathcal{N}_f)$

(Glaser,Surya arXiv:1410.8775)

How to measure this in MCMC?

$$\Psi_0^{(\beta)}(\mathcal{N}_f) = A\mathcal{Z}_\beta(\mathcal{N}_f)$$

Using statistical mechanics

$$\langle \mathcal{S}_{2\mathrm{d}} \rangle(\beta) = \frac{\partial \ln \mathcal{Z}_\beta(\mathcal{N}_f)}{\partial \beta}$$

We can then define

$$A\mathcal{Z}_\beta(\mathcal{N}_f) = A\mathcal{Z}_0(\mathcal{N}_f)\, e^{-\int_0^\beta d\beta' \langle \mathcal{S}_{2\mathrm{d}} \rangle(\beta')}$$

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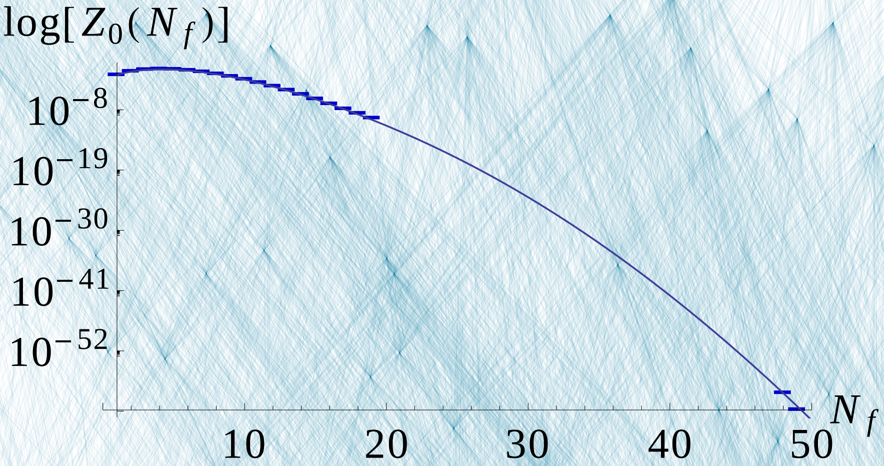
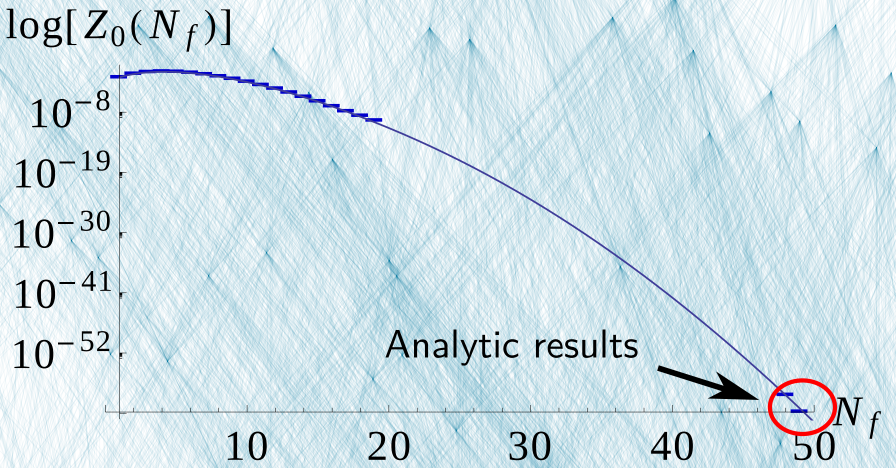
Concrete implementation

- measure $\langle \mathcal{S}_{2\mathrm{d}} \rangle(\beta')$ for fixed $\mathcal{N}_f$ using MCMC
- calculate normalization $\mathcal{Z}_0(\mathcal{N}_f)$

$$\mathcal{Z}_0(\text{free}) = \sum_{\mathcal{N}_f} \mathcal{Z}_0(\mathcal{N}_f)$$

- $\mathcal{Z}_0(\text{free})$ is the ensemble of all 2d orders
- The relative frequency for orders with $\mathcal{N}_f$ final elements in $\mathcal{Z}_0(\text{free})$ does then give us $\mathcal{Z}_0(\mathcal{N}_f)$ up to an overall constant


$\log[Z_0(N_f)]$
10^{-8}
10^{-19}
10^{-30}
10^{-41}
10^{-52}
10
20
30
40
50
N_f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10^{-8}
10^{-19}
10^{-30}
10^{-41}
10^{-52}
Analytic results
N_f
10
20
30
40
50

$\langle S(\beta, \mathcal{N}_f) \rangle$ for all $\mathcal{N}_f$

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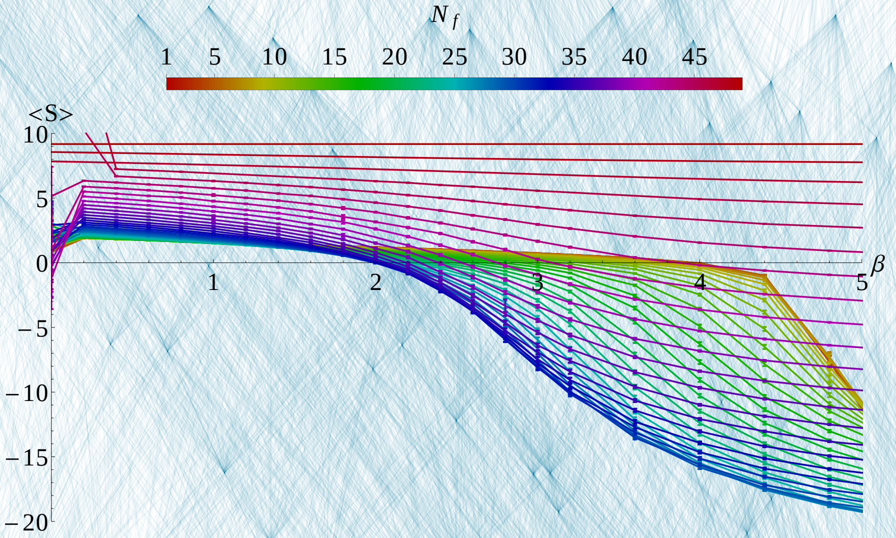
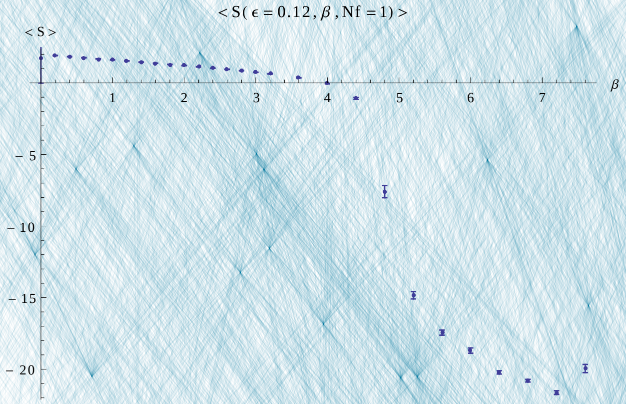


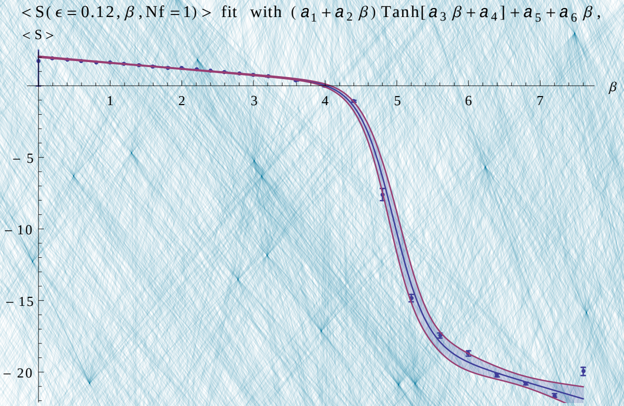
How do we integrate $\langle \mathcal{S}_{2\mathrm{d}} \rangle$?

Measure $\langle S \rangle$ for different β



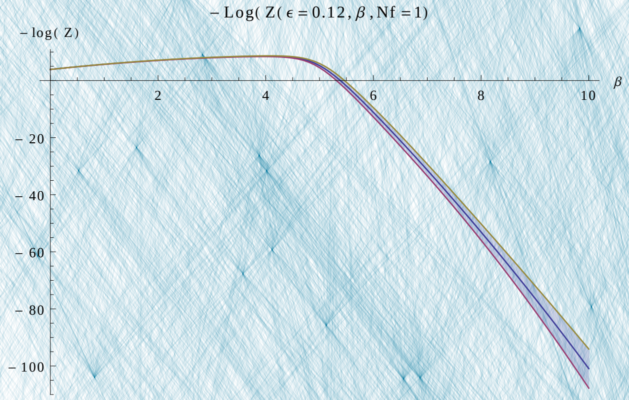
How do we integrate $\langle \mathcal{S}_{2\mathrm{d}} \rangle$?

Fit a function to the data

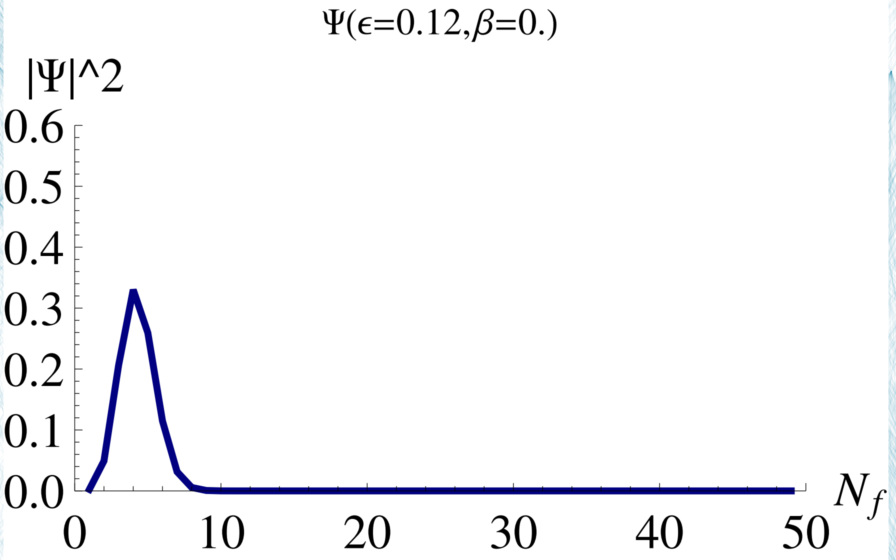


How do we integrate $\langle \mathcal{S}_{2\mathrm{d}} \rangle$?

Integrate the fit function for an estimate of $-\log Z$



The wave function



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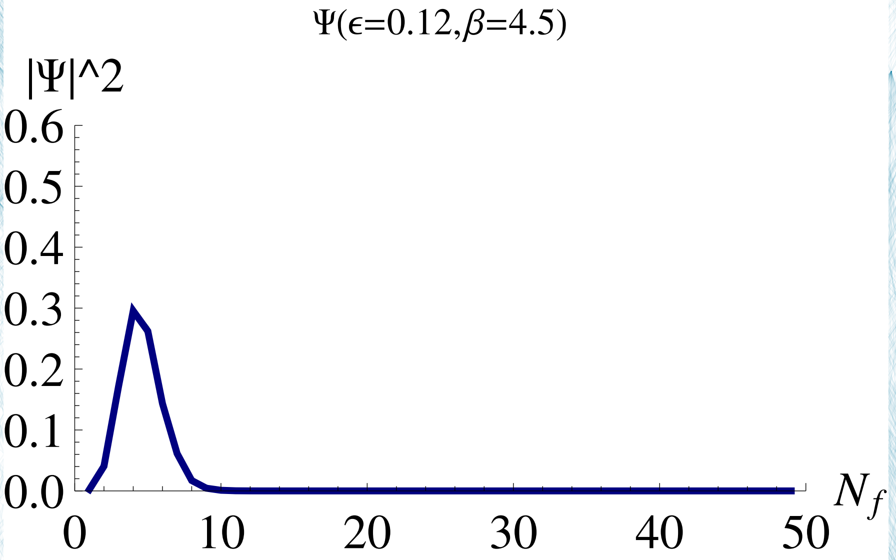

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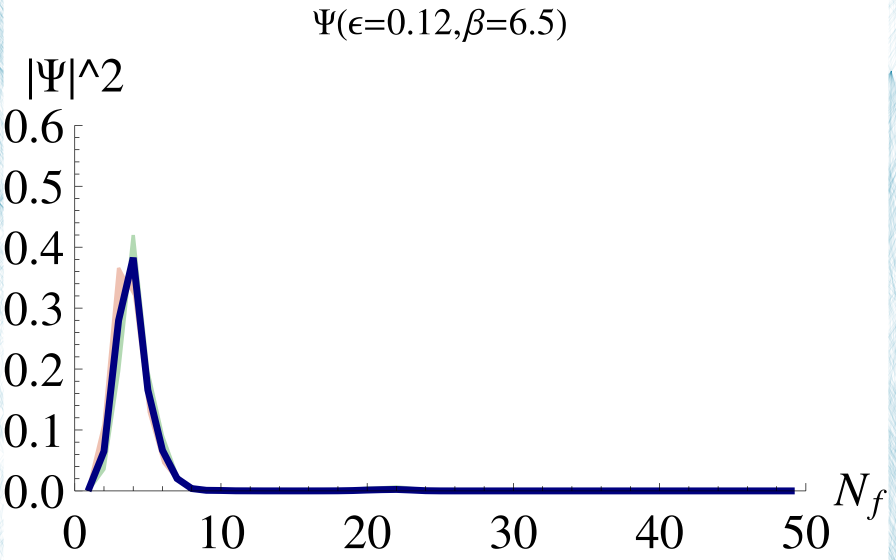

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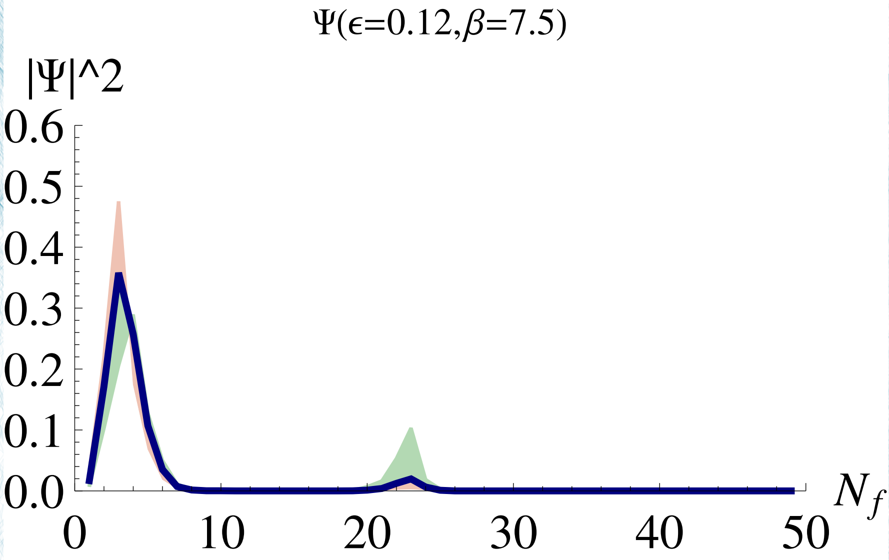

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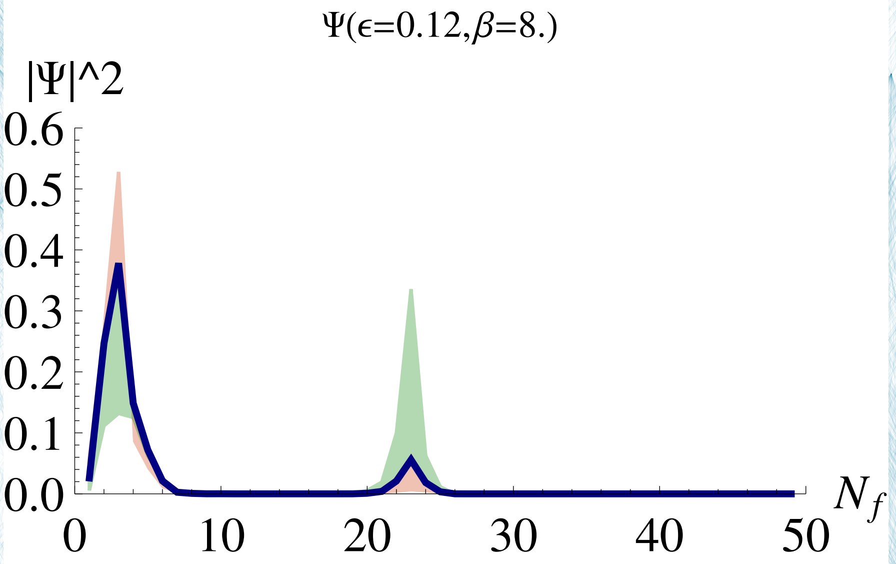

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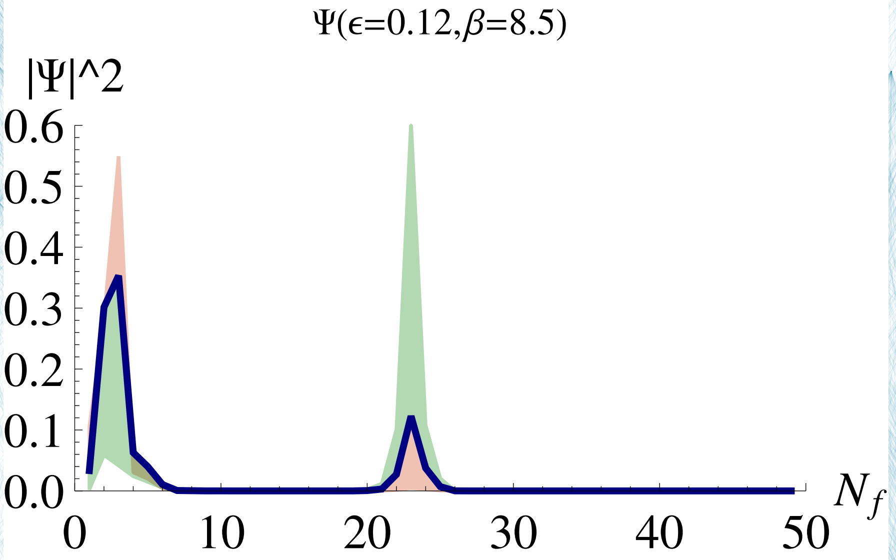

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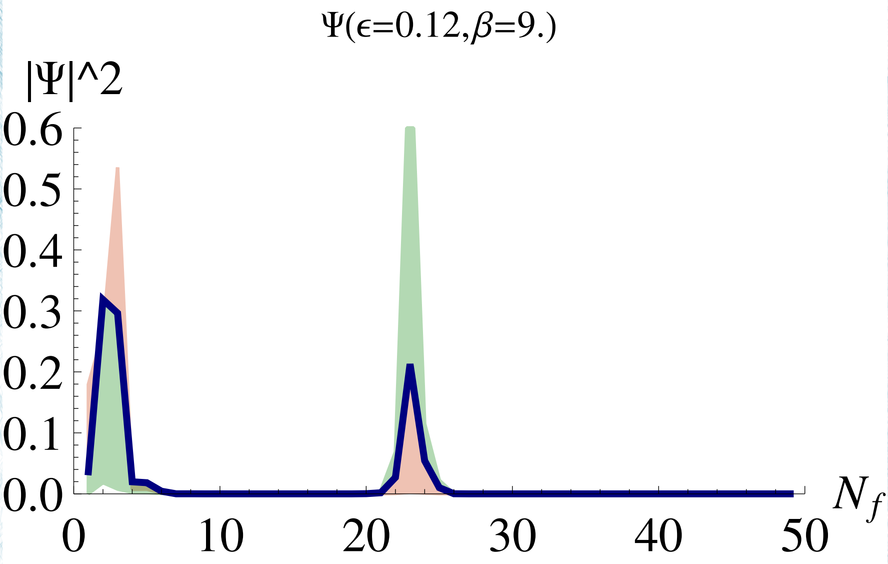

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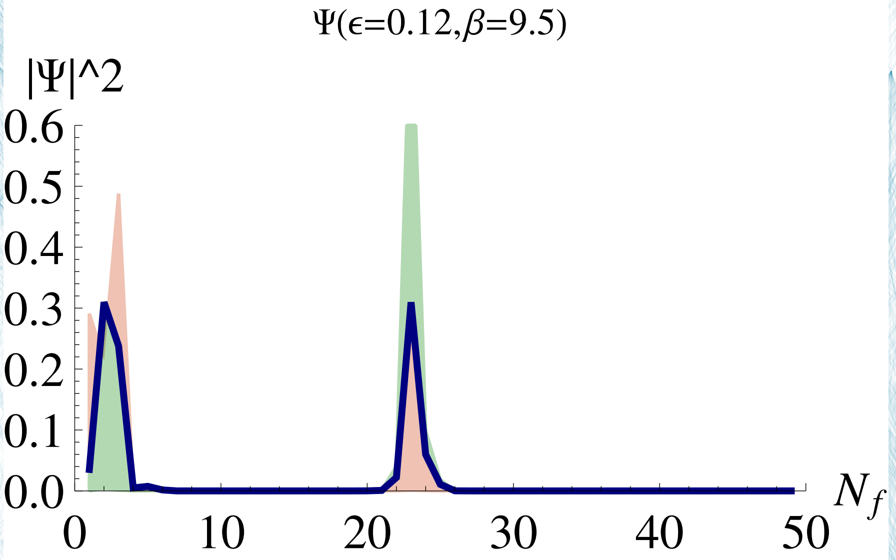

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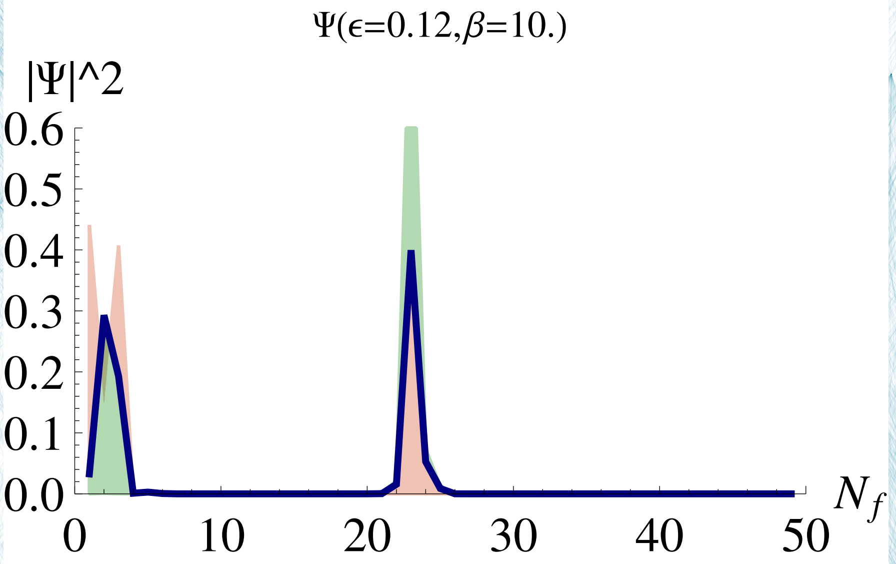
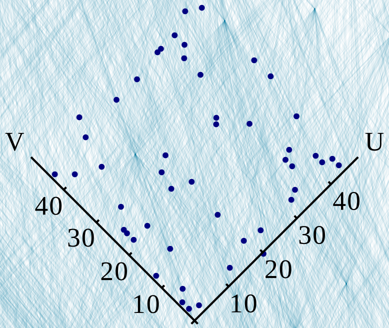


Geometry in the 1st peak (low β)

- Continuum type 2-d order
- Dominated by $\mathcal{Z}_0$
- ‘as high as wide’



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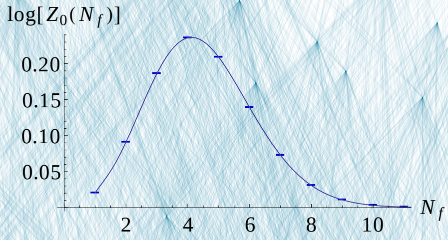

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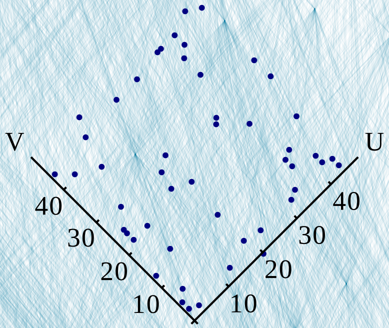

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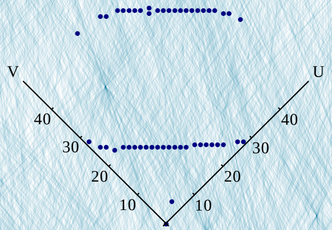
- Crystalline structure
- Fast expansion
- Homogeneous pasts



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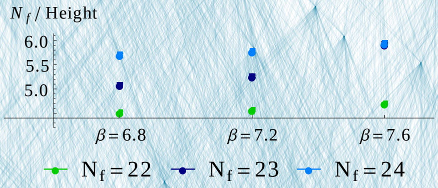

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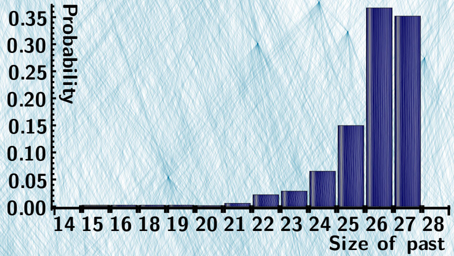

Summary & Conclusion

What did we do?

- 2d orders
- transition from single point to $\mathcal{N}_f$

What did we find

Two phases

- high temperature (low β): continuum like phase
- low temperature (high β): rapidly expanding, crystalline phase

Thank you for your attention!

What did we do?

- 2d orders
- transition from single point to $\mathcal{N}_f$

What did we find

Two phases

- high temperature (low β): continuum like phase
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What is Ω_{2d} exactly?

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Ω_{2d} is the set of N -element “2D orders”

Definition

Let $S = (1, ..., N)$ and $U = (u_1, u_2, ..., u_N)$, $V = (v_1, v_2, ..., v_N)$, with $u_i, v_i \in S$. U and V are then total orders with $\prec$ given by the natural ordering $<$ in S.
An N -element 2D order is the intersection $C = U \cap V$ of two total N-element orders U and V , i.e., $e_i \prec e_j$ in C iff $u_i < u_j$ and $v_i < v_j$.

This corresponds to lightcone coordinates.

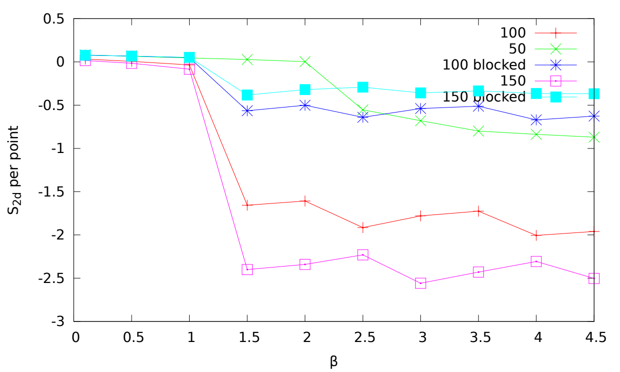
Back

Open questions on the HH wave function

- recently introduced boundary term
- full simulation without restriction to 2d orders

Open questions in 2d MC

- How does the phase transition behave for other volumes ?
- Does the random phase carry over to negative β^2, the quantum theory?



(Glaser,Surya, ongoing work)