On the role of the extra kinetic term coupling in Hořava-Lifshitz gravity

because a small title would have been too nice

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Hořava-Lifshitz gravity (HLG)

An attempt to build a perturbatively renormalizable theory of gravity valid at all scales. Amongst its properties:

- built in unitarity
- no extra fields when compared with GR,
- no extra dimensions.

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Due to the lack of free lunches which characterizes life, the universe and everything, all these nice properties come at a price:

- Lorentz invariance is broken,
- It is far from clear if GR can be recovered in the IR.

Outline

HLG I

Let us define the theory in the following three steps:

- Propose a non Gaussian UV fixed point (UV FP),
- Choose symmetry group which respects the properties of the UV FP,
- Construct the most general action respecting the symmetries and being such that:
  - there are no more than two time derivatives so as to get unitarity,
  - in terms of power counting, it is perturbatively renormalizable.

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Step 0: field content

There are two options regarding the field content:

- Insist in matching that of GR $\rightarrow$ $N(x,t)$, $N^i(x,t)$, $g_{ij}(x,t)$.
  - This is the so-called non-projectable HLG.
- Consider only $g_{ij}(x,t)$ as fundamental and add only what is strictly necessary to build invariants.
  - This leads to projectable HLG: same as before but with $N \equiv N(t)$.

HLG II - The UV fixed point

The UV FP is such that solutions of the theory at that scale should satisfy the anisotropic scaling

$$t \to b^z t, \qquad\qquad x^i \to b x^i.$$

- z - critical exponent characterizing the theory,
- $z \neq 1 \;\Rightarrow$ preferred notion of time.

Due to the preffered notion of time, 4-dimensional Diffeomorphisms are not an appropriate symmetry for these theories!

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Manifold restrictions

To have a preferred notion of time, consider only manifolds admitting a global time foliation,

$$\mathcal{M} = \mathbb{R} \times \Sigma$$

- Here Σ will be closed and compact unless otherwise specified.

HLG III - Replacing diffeomorphism invariance

Accepting we cannot have invariance under 4-dimensional diffeomorphisms, what we want is:

- a symmetry group which preserves the foliation structure,
- a symmetry group as close to diffeomorphisms as possible.

Choice: impose invariance under foliation-preserving Diffeomorphisms.

- Infinitesimal generators of $\mathrm{Diff}_{\mathcal{F}}(\mathcal{M})$

$$\delta t = f(t), \qquad \delta x^i = \zeta^i(x,t)$$

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Extrinsic curvature

Like in GR, one needs K_{ij} to build invariants with time derivatives of the metric,

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right),$$

- Unlike in GR, this can just as well be done with $N \equiv N(t)$, hence the existence of the projectable version.

HLG IV - Kinetic term of the action

Because of the reduced symmetry group, both $K_{ij}K^{ij}$ and K^2 are independently invariant,

$$S_K = \frac{1}{g_\kappa}\int dt \int d^3x \sqrt{g}\, N\left(K_{ij}K^{ij} - \lambda K^2\right) = \frac{1}{g_\kappa}\int dt \int d^3x \sqrt{g}\, N K_{ij} G_\lambda^{ijkl} K_{kl},$$

with "little lambda" λ a new dimensionless coupling.

- $\lambda = 1$ restores the Kinetic term of GR,
- G_λ^{ijkl} is a generalized Wheeler-DeWitt metric,

  $$G_\lambda^{ijkl} = \frac{1}{2}\left(g^{ik}g^{jl} + g^{il}g^{jk}\right) - \lambda g^{ij}g^{kl},$$

  which is invertible only for $\lambda \neq \frac{1}{3}$,

  $$G^\lambda_{ijkl} = \frac{1}{2}\left(g_{ik}g_{jl} + g_{il}g_{jk}\right) - \frac{\lambda}{3\lambda - 1} g_{ij}g_{kl}.$$

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Picking a z

As it turns out, in $d+1$ dimensions, demanding $[g_\kappa] = 0$ determines z to be $z = d$.

HLG V - Potential term

The potential term of the action then contains all invariants containing only spatial derivatives,

$$S_V = \int dt \int d^3x \sqrt{g}\, N\mathcal{V}(g_{ij}, N)$$

- Since $\left[K^2\right] = 2d$, for 3 spatial dimensions, terms with up to 6 spatial derivatives are allowed
- Examples of terms that should appear include R^2, $RR_{ij}R^{ij}$, $g^{kl}\nabla_k R_{ij} \nabla_l R^{ij}$, among many others.

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An extra invariant

For the non-projectable version of the theory, an extra class of invariants should be included:

- As it turns out, $a_i \equiv \nabla_i \log N$ transforms as a vector under $\mathrm{Diff}_{\mathcal{F}}(\mathcal{M})$,
- They imply a proliferation of invariants on the potential term of the action
    - because it was not messy enough as it was.

Classical limit and the λ-R model

The action describing the low energy limit of the theory is then

$$S = \int dt \int d^3x \sqrt{g}\, N \left(K_{ij} G_\lambda^{ijkl} K_{kl} + R - 2\Lambda + \beta a_i a^i \right).$$

Because our goal is to understand the role of λ, we will set $\beta = 0$.

- The system thus described will be referred to as a λ-R model.
- In this case, both versions of the theory are described by the same action,
- The different results for both of them illustrate the role of the projectability condition.

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Goal

To give an answer to the question:

- is it really necessary for $\lambda \to 1$ for a λ-R model to reproduce GR?

Non-projectable HLG I - Legendre transformation

Conjugate momenta

We start by defining generalized momenta in the usual way,

$$\pi^{ij} \equiv \frac{\delta S}{\delta \dot{g}^{ij}} = \sqrt{g} G_\lambda^{ijkl} K_{kl}$$

$$\phi \equiv \frac{\delta S}{\delta \dot{N}} = 0, \qquad \phi_i \equiv \frac{\delta S}{\delta \dot{N}^i} = 0$$

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One can then perform the Legendre transformation to obtain the total Hamiltonian,

$$H_t = \int d^3x \left\{ N\mathcal{H} + N^i \mathcal{H}_i + \alpha\phi + \alpha^i \phi_i \right\},$$

with α and α^i being Lagrange multipliers and $\mathcal{H}$ and $\mathcal{H}_i$ given by

$$\mathcal{H} \equiv \frac{G^{\lambda}_{ijkl}}{\sqrt{g}} \pi^{ij} \pi^{kl} - \sqrt{g}\,(R - 2\Lambda)\,,$$

$$\mathcal{H}_i \equiv -2 g_{ij} \nabla_k \pi^{jk}.$$

Non-projectable HLG II - Constraints

- Since there are no time derivatives of N and N^i on the action, their momenta define the four primary constraints of the theory

$$\phi = 0, \qquad \phi_i = 0,$$

- Their time preservation yield the familiar looking (modified) Hamiltonian and momentum constraints,

$$\dot{\phi} = \mathcal{H} \approx 0, \qquad \dot{\phi}_i = \mathcal{H}_i \approx 0.$$

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Some things change, some stay the same

We now need to impose $\dot{\mathcal{H}} \approx 0$ and $\dot{\mathcal{H}}_i \approx 0$.

- Since neither $\mathcal{H}_i$ nor spatial diffeomorphisms changed, all PBs involving $\mathcal{H}_i$ still vanish on the constraint surface.
- Due to λ, the PB between $\mathcal{H}$ and itself contains an extra term,
  - this extra term is exclusively present on the non-projectable theory,
  - because of its presence, $\dot{\mathcal{H}} \approx 0$ implies a tertiary constraint.

Non-projectable HLG III - The tertiary constraint

Tertiary constraint

- From imposing $\dot{\mathcal{H}} \approx 0$ we obtain
$$\dot{\mathcal{H}} = -2\frac{1-\lambda}{3\lambda-1}\left(N\nabla^2\pi + 2g^{ij}\nabla_i\pi\nabla_j N\right) \approx 0$$
- **Remark**: the non-trivial term comes from the $\{\mathcal{H},\mathcal{H}\}$ part of $\dot{\mathcal{H}}$,
  - more precisely, from the last term in the variation of R:
$$\delta_{g_{ij}}\int d^3x N\sqrt{g}R = \sqrt{g}N\left(\frac{1}{2}g^{ij}R - R^{ij}\right) + \sqrt{g}G_1^{ijkl}\nabla_k\nabla_l N$$
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Asymptotically flat spacetime

Note that the only solution of (1) for asymptotically flat spacetimes is $\pi = 0$. In this case all the λ-dependence drops out and GR is recovered.

NP HLG IV - Tertiary and quaternary constraints

For compact hypersurfaces, solutions to (1) are given by the constant mean curvature (CMC) gauge condition

$$\dot{\mathcal{H}} \approx 0 \Rightarrow \omega \equiv \pi - a(t)\sqrt{g} \approx 0, \quad (2)$$

- $a(t)$ can be any spatial constant,
  - In our original papper, we considered $a(t) = 0$,
  - Here, we use a more general solution obtained by integrating (2) over Σ,

$$a(t) = \frac{1}{V} \int d^3x\pi.$$

- since π does not vanish, a non-trivial quaternary constraint emerges.

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Quaternary constraint (I promise it's the last one)

The quaternary constraint is a λ-dependent Lapse fixing equation

$$\mathcal{M} \equiv D_\lambda N - \frac{\sqrt{g}}{V}\int d^3x D_\lambda N \approx 0,$$

$$D_\lambda \equiv \sqrt{g}\left(R - 3\Lambda + \frac{a^2}{2(3\lambda - 1)} - \nabla^2\right).$$

NP HLG V - Closing the algebra and counting d.o.f.

- To wrap up the constraint algebra, one must check the time preservation of $\mathcal{M} \approx 0$.
- Thankfully, that determines the Lagrange multiplier α associated with $\phi \approx 0$,

$$F + D_\lambda \alpha - \frac{\sqrt{g}}{V} \int d^3x \, (F + D_\lambda \alpha) \approx 0,$$

$$F = \left(2\pi^{kl} - \pi g^{kl} \frac{2\lambda-1}{3\lambda-1}\right) \left(N \nabla_k \nabla_k N + \nabla_k \left(N \nabla_l N\right) - N^2 R_{kl}\right) + \frac{N^2 \pi R}{3\lambda-1} - \frac{\alpha N D_\lambda N}{3\lambda-1}$$

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Degree of freedom counting

- Upon redefining $\mathcal{H}_i$ to include the action of spatial diffeomorphisms on N and ϕ, we are left with:
  - 6 first class constraints, $\mathcal{H}_i \approx 0$ and $\phi^i \approx 0$
  - 4 second class constraints, $\mathcal{H} \approx 0$, $\omega \approx 0$, $\mathcal{M} \approx 0$, and $\phi \approx 0$.
- Despite the presence of λ on the e.o.m., $\mathcal{H} \approx 0$, and $\mathcal{M} \approx 0$, the # of local physical d.o.f. is the same as in GR - two.

NP HLG VI - momentum decomposition

Whatever we have, we know that:

- We have a one-to-one correspondence between our constraints and all consistency conditions present in GR with the CMC gauge,
- We have exactly the same number of physical degrees of freedom,

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One unpleasant feature is that λ appears in different ways in $\mathcal{M} \approx 0$ and $\dot{\mathcal{M}} \approx 0$. Here's a way to fix that:

Momentum decomposition

Without loss of generality, decompose the momentum tensor density as

$$\pi^{ij} = \pi^{ij}_{TT} + \nabla^i u^j + \nabla^j u^i + \frac{1}{3} g^{ij}\pi. \quad (3)$$

Plug (3) in $\mathcal{H}_i \approx 0$ and notice the follow solves the constraint

$$\pi^{ij} = \pi^{ij}_{TT} + \frac{1}{3} g^{ij}\pi. \quad (4)$$

(4) not only solves $\mathcal{H}_i \approx 0$ but turns all λ-dependence into $\frac{1}{3\lambda - 1}$.

NP HLG VII - modified Lichnerowicz-York equation

- Another advantage of the TT decomposition, is that it is the first step towards the Lichnerwoicz-York equation in GR,
- The second step would be assuming $\nabla_i \pi = 0$, which we have for free!
- Consider the following conformal transformation,

$$\bar{g}_{ij} = \phi^4 g_{ij}, \qquad \bar{\pi}^{ij}_{TT} = \phi^{-4} \phi^{ij}_{TT}, \qquad \bar{\pi} = \phi^6 \pi. \qquad (5)$$

- Write down $\mathcal{H} \approx 0$ for the barred variables. Plug (5) and obtain an equation for ϕ, a modified Lichnerowicz-York equation,

$$8\nabla^2 \phi - R\phi + \phi^{-7} \frac{\pi^{ij}_{TT} \pi_{ij}^{TT}}{g} - \frac{\phi^5}{3(3\lambda - 1)} \frac{\pi^2}{g} \approx 0. \quad (6)$$

- If $\lambda > 1/3$, we are guaranteed that there is a $\phi > 0$ solving (6) and it is unique.

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- If $\lambda > 1/3$, we are guaranteed that there is a $\phi > 0$ solving (6) and it is unique.

So, is this GR or not? The usual initial value techniques for GR don't seem to generalize for $\lambda < 1/3$. For $\lambda > 1/3$ things work but we clearly get a different ϕ.

Conclusion

- Upon a careful examination of the constraint algebra we have:
  - The same number of local d.o.f. found in GR,
  - The CMC gauge condition is naturally imposed on the theory,
- After a TT decomposition, we obtained a modified LY equation,
  - The λ-dependence becomes the same everywhere and $\mathcal{H}_i \approx 0$ is solved,
  - The modified LY equation has unique solutions for $\lambda > 1/3$,
  - For an arbitrary base metric, the conformal factor solving $\mathcal{H} \approx 0$ is not the same as in GR.

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Thank you!
Don't make a sound: they're not dead, just sleeping.