An Introduction to Shape Dynamics

Sean Gryb

Radboud Universiteit Nijmegen
Institute for Mathematics, Astrophysics
and Particle Physics

Quantum Gravity in Cracow[4]
10 May 2015

Introduction

Conformal Symmetry in Gravity?

Conformal Symmetry in Gravity?

Why?

1. No dimensionful quantities.

Conformal Symmetry in Gravity?

Why?

1. No dimensionful quantities.
2. UV fixed point $\Rightarrow$ conformal.

Shape Dynamics

Claim

∃ foliation where gauge-invariant observables of GR are 3d conformally (Weyl) invariant.

$$g_{ab}^{(3)} \rightarrow e^{\phi(x)} g_{ab}^{(3)}$$

Shape Dynamics

Claim

$\exists$ foliation where gauge-invariant observables of GR are 3d conformally (Weyl) invariant.

$$g_{ab}^{(3)} \to e^{\phi(x)} g_{ab}^{(3)}$$

Caveat: IR is (nearly) CONFORMAL. (But non-local!)

Shape Dynamics

Claim

$\exists$ foliation where gauge-invariant observables of GR are 3d conformally (Weyl) invariant.

$$g^{(3)}_{ab} \rightarrow e^{\phi(x)} g^{(3)}_{ab}$$

Caveat: IR is (nearly) CONFORMAL. (But non-local!)

$\Rightarrow$ **Ontological Shift:** Only conformal degrees of freedom are fundamental!

Shape Dynamics

Claim

$\exists$ foliation where gauge-invariant observables of GR are 3d conformally (Weyl) invariant.

$$g_{ab}^{(3)} \to e^{\phi(x)} g_{ab}^{(3)}$$

Caveat: IR is (nearly) CONFORMAL. (But non-local!)

$\Rightarrow$ **Ontological Shift:** Only conformal degrees of freedom are fundamental!

Ignoring scale can lead to important global differences.

Shape Dynamics Basics

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_{\mathrm{e}}(g_{ab},\phi;\pi^{ab},\pi_{\phi})$

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_{\mathrm{e}}(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi} g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4 g_{ab}\pi^{ab}$

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_{\mathrm{e}}(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi}g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4g_{ab}\pi^{ab}$.

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_{\mathrm{e}}(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi} g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4 g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi} g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi} g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4 g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi} g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4 g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi} g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi} g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4 g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.

The Road to Shape Dynamics

- ADM $\to$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \to \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \to e^{4\phi} g_{ab}$, $\pi^{ab} \to e^{-4\phi}\pi^{ab}$, $\phi \to \phi$, $\pi_\phi \to \pi_\phi - 4 g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4 g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.
- Linking Theory $\to$ ADM
  $\Rightarrow$ Gauge fixing $\phi = 0$ of $C = 0$.

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi}g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.
- Linking Theory $\rightarrow$ ADM
  $\Rightarrow$ Gauge fixing $\phi = 0$ of $C = 0$.
- Linking Theory $\rightarrow$ Shape Dynamics
  - Gauge fixing $\pi_\phi = 0$ of $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab}) = 0$

The Road to Shape Dynamics

- ADM $\to$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \to \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \to e^{4\phi} g_{ab}$, $\pi^{ab} \to e^{-4\phi}\pi^{ab}$, $\phi \to \phi$, $\pi_\phi \to \pi_\phi - 4 g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.
- Linking Theory $\to$ ADM
  $\Rightarrow$ Gauge fixing $\phi = 0$ of $C = 0$.
- Linking Theory $\to$ Shape Dynamics
  - Gauge fixing $\pi_\phi = 0$ of $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab}) = 0$
  - Solve Hamiltonian constraint for conformal factor (elliptic equation).

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi}g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.
- Linking Theory $\rightarrow$ ADM
  $\Rightarrow$ Gauge fixing $\phi = 0$ of $C = 0$.
- Linking Theory $\rightarrow$ Shape Dynamics
  - Gauge fixing $\pi_\phi = 0$ of $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab}) = 0$
  - Solve Hamiltonian constraint for conformal factor (elliptic equation).
  - $\therefore$ fixes foliation in Linking Theory.

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi} g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4 g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4 g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.
- Linking Theory $\rightarrow$ ADM

  $\Rightarrow$ Gauge fixing $\phi = 0$ of $C = 0$.
- Linking Theory $\rightarrow$ Shape Dynamics
  - Gauge fixing $\pi_\phi = 0$ of $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab}) = 0$
  - Solve Hamiltonian constraint for conformal factor (elliptic equation).
  - $\therefore$ fixes foliation in Linking Theory.

  $\Rightarrow$ $C = 0$ becomes $g_{ab}\pi^{ab} = 0$ (generator of conformal transformations)!

The Road to Shape Dynamics

- ADM $\rightarrow$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \rightarrow \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \rightarrow e^{4\phi} g_{ab}$, $\pi^{ab} \rightarrow e^{-4\phi}\pi^{ab}$, $\phi \rightarrow \phi$, $\pi_\phi \rightarrow \pi_\phi - 4 g_{ab}\pi^{ab}$

  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.
- Linking Theory $\rightarrow$ ADM

  $\Rightarrow$ Gauge fixing $\phi = 0$ of $C = 0$.
- Linking Theory $\rightarrow$ Shape Dynamics
  - Gauge fixing $\pi_\phi = 0$ of $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab}) = 0$
  - Solve Hamiltonian constraint for conformal factor (elliptic equation).
  - $\therefore$ fixes foliation in Linking Theory.

  $\Rightarrow$ $C = 0$ becomes $g_{ab}\pi^{ab} = 0$ (generator of conformal transformations)!
- Shape Dynamics $\rightarrow$ Dictionary (ADM in CMC)

  $\Rightarrow$ $\mathcal{H}(g_{ab},\pi^{ab}) = 0$ gauge fixes $\pi = 0$ (elliptic equation).

The Road to Shape Dynamics

- ADM $\to$ Linking Theory
  1. Phase space extension: $\Gamma(g_{ab};\pi^{ab}) \to \Gamma_e(g_{ab},\phi;\pi^{ab},\pi_\phi)$
  2. Canonical transformation: $g_{ab} \to e^{4\phi} g_{ab}$, $\pi^{ab} \to e^{-4\phi}\pi^{ab}$, $\phi \to \phi$, $\pi_\phi \to \pi_\phi - 4 g_{ab}\pi^{ab}$

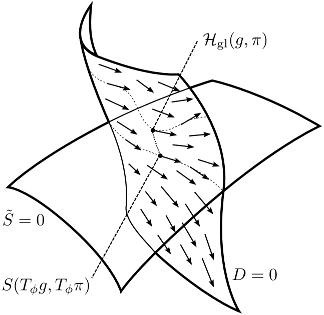
  - Constraints: $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $\mathcal{H}_a(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab})$, $C \equiv \pi_\phi - 4g_{ab}\pi^{ab}$.
  - Interpretation: Conformally compensated ADM.
  - C generates canonical transformation.
- Linking Theory $\to$ ADM
  $\Rightarrow$ Gauge fixing $\phi = 0$ of $C = 0$.
- Linking Theory $\to$ Shape Dynamics
  - Gauge fixing $\pi_\phi = 0$ of $\mathcal{H}(e^{4\phi}g_{ab}, e^{-4\phi}\pi^{ab}) = 0$
  - Solve Hamiltonian constraint for conformal factor (elliptic equation).
  - $\therefore$ fixes foliation in Linking Theory.

  $\Rightarrow$ $C = 0$ becomes $g_{ab}\pi^{ab} = 0$ (generator of conformal transformations)!
- Shape Dynamics $\to$ Dictionary (ADM in CMC)
  $\Rightarrow$ $\mathcal{H}(g_{ab},\pi^{ab}) = 0$ gauge fixes $\pi = 0$ (elliptic equation).

$\therefore$ a conformal gauge of Shape Dynamics is equivalent to a foliation of ADM!

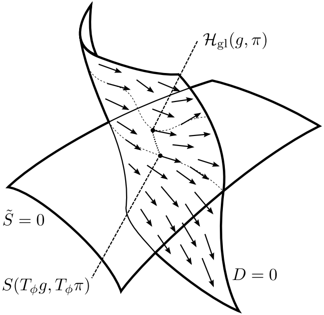
Overall Picture

Shape Dynamics and ADM share a common gauge



Overall Picture

Shape Dynamics and ADM share a common gauge

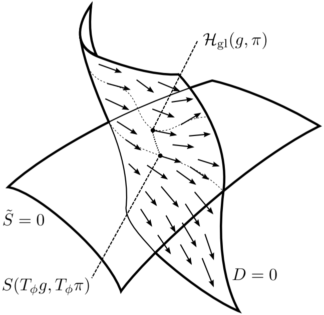


Take home messages:

- Shape Dynamics is constructed by solving two elliptic differential equations for ϕ ($\mathcal{H}$ gauge fixing) and N (propagation of $g_{ab}\pi^{ab}=0$).

Overall Picture

Shape Dynamics and ADM share a common gauge

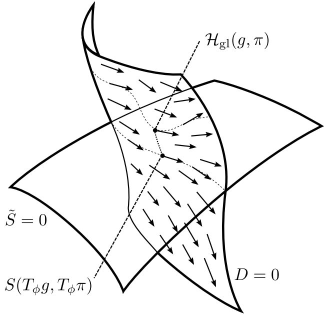


Take home messages:

- Shape Dynamics is constructed by solving two elliptic differential equations for ϕ ($\mathcal{H}$ gauge fixing) and N (propagation of $g_{ab}\pi^{ab}=0$).
- A non-trivial SD Hamiltonian is only obtained by putting a global restriction on ϕ:
  - For $\partial\Sigma = 0$ topology: $\text{vol}(g_{ab}) = \text{vol}(e^{-4\phi} g_{ab})$.

Overall Picture

Shape Dynamics and ADM share a common gauge

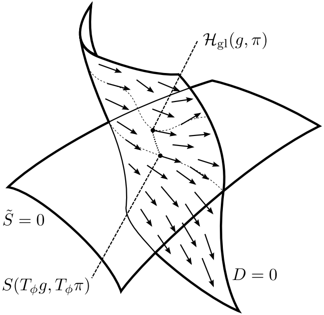


Take home messages:

- Shape Dynamics is constructed by solving two elliptic differential equations for ϕ ($\mathcal{H}$ gauge fixing) and N (propagation of $g_{ab}\pi^{ab}=0$).
- A non-trivial SD Hamiltonian is only obtained by putting a global restriction on ϕ:
  - For $\partial\Sigma = 0$ topology: $\text{vol}(g_{ab}) = \text{vol}(e^{-4\phi} g_{ab})$.
  - For open Σ: fall-off conditions on ϕ (e.g., asymptotically flat)

Overall Picture

Shape Dynamics and ADM share a common gauge



Take home messages:

- Shape Dynamics is constructed by solving two elliptic differential equations for ϕ ($\mathcal{H}$ gauge fixing) and N (propagation of $g_{ab}\pi^{ab}=0$).
- A non-trivial SD Hamiltonian is only obtained by putting a global restriction on ϕ:
  - For $\partial\Sigma = 0$ topology: $\mathrm{vol}(g_{ab}) = \mathrm{vol}(e^{-4\phi} g_{ab})$.
  - For open Σ: fall-off conditions on ϕ (e.g., asymptotically flat)

$\Rightarrow$ dimensional constants are smuggled in!

Classical Consequences

Consequences: Classical

Locally Shape Dynamics is equivalent to GR.

Consequences: Classical

Locally Shape Dynamics is equivalent to GR.

but

Globally solutions of both theories can differ!

Consequences: Classical

Locally Shape Dynamics is equivalent to GR.

but

Globally solutions of both theories can differ!

Examples

- Kruskal $\Rightarrow$ (Traversable) Einstein–Rosen bridge.
  $\Rightarrow$ singularity containing region is excluded!

Consequences: Classical

Locally Shape Dynamics is equivalent to GR.

but

Globally solutions of both theories can differ!

Examples

- Kruskal $\Rightarrow$ (Traversable) Einstein–Rosen bridge.
  $\Rightarrow$ singularity containing region is excluded!
- N-body dynamics projected on shape space is dissipative.
  $\Rightarrow$ Arrow of Time.

Consequences: Classical

Locally Shape Dynamics is equivalent to GR.

but

Globally solutions of both theories can differ!

Examples

- Kruskal $\Rightarrow$ (Traversable) Einstein–Rosen bridge.
  $\Rightarrow$ singularity containing region is excluded!
- N-body dynamics projected on shape space is dissipative.
  $\Rightarrow$ Arrow of Time.
- Others??

Spherically Symmetric Shape Dynamics

Spherically Symmetric Ansatz

General *spacetime* Ansatz:

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + A^2 \mathrm{d}r^2 + B^2 \mathrm{d}\Omega^2 \,.$$

If we choose (coordinate transformation)

$$A = e^{2\phi} \qquad\qquad B = r e^{2\phi} \tag{1}$$

Then

$$g^{(3)}_{ab} = e^{4\phi(x)} \mathrm{Diag}(1, r^2, r^2 \sin^2\theta)_{ab}$$

Spherically Symmetric Shape Dynamics

Spherically Symmetric Ansatz

General *spacetime* Ansatz:

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + A^2\mathrm{d}r^2 + B^2\mathrm{d}\Omega^2 \,.$$

If we choose (coordinate transformation)

$$A = e^{2\phi} \qquad B = re^{2\phi} \tag{1}$$

Then

$$g^{(3)}_{ab} = e^{4\phi(x)}\mathrm{Diag}(1, r^2, r^2 \sin^2\theta)_{ab}$$

$\Rightarrow$ general spherically symmetric spatial metric is conformally flat!

Spherically Symmetric Shape Dynamics

Spherically Symmetric Ansatz

General *spacetime* Ansatz:

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + A^2\mathrm{d}r^2 + B^2\mathrm{d}\Omega^2 \,.$$

If we choose (coordinate transformation)

$$A = e^{2\phi} \qquad\qquad B = re^{2\phi} \tag{1}$$

Then

$$g^{(3)}_{ab} = e^{4\phi(x)}\mathrm{Diag}(1, r^2, r^2\sin^2\theta)_{ab}$$

$\Rightarrow$ general spherically symmetric spatial metric is conformally flat!

But: ϕ is gauge in Shape Dynamics!

Spherically Symmetric Shape Dynamics

Spherically Symmetric Ansatz

General *spacetime* Ansatz:

$$ds^2 = -N^2 dt^2 + A^2 dr^2 + B^2 d\Omega^2 \,.$$

If we choose (coordinate transformation)

$$A = e^{2\phi} \qquad\qquad B = re^{2\phi} \tag{1}$$

Then

$$g^{(3)}_{ab} = e^{4\phi(x)} \mathrm{Diag}(1, r^2, r^2 \sin^2\theta)_{ab}$$

$\Rightarrow$ general spherically symmetric spatial metric is conformally flat!

But: ϕ is gauge in Shape Dynamics!

$\boxed{\Rightarrow \text{Spherically Symmetric SD is } \textit{static} \text{ conformal geometry!}}$

Global Parameters

Integration constants for (N, ϕ) in SD Hamiltonian?

Global Parameters

Integration constants for (N, ϕ) in SD Hamiltonian?

Interpretation

- Integration constants $\Rightarrow$ affect Hamiltonian.

Global Parameters

Integration constants for (N, ϕ) in SD Hamiltonian?

Interpretation

- Integration constants $\Rightarrow$ affect Hamiltonian.
- Test matter fields 'feel' integration constants through minimal coupling to SD Hamiltonian.

Global Parameters

Integration constants for (N, ϕ) in SD Hamiltonian?

Interpretation

- Integration constants $\Rightarrow$ affect Hamiltonian.
- Test matter fields ‘feel’ integration constants through minimal coupling to SD Hamiltonian.

$\therefore$ integration constants parametrize propagation of test fields in SD.

Global Parameters

Integration constants for (N, ϕ) in SD Hamiltonian?

Interpretation

- Integration constants $\Rightarrow$ affect Hamiltonian.
- Test matter fields 'feel' integration constants through minimal coupling to SD Hamiltonian.

$\therefore$ integration constants parametrize propagation of test fields in SD.

Methodology

(At least) 2 perspectives:

1. (Rigorous) Minimally couple matter to SD, take zero back-reaction limit, analyse propagation of fields.

Global Parameters

Integration constants for (N, ϕ) in SD Hamiltonian?

Interpretation

- Integration constants $\Rightarrow$ affect Hamiltonian.
- Test matter fields 'feel' integration constants through minimal coupling to SD Hamiltonian.

$\therefore$ integration constants parametrize propagation of test fields in SD.

Methodology

(At least) 2 perspectives:

1. (Rigorous) Minimally couple matter to SD, take zero back-reaction limit, analyse propagation of fields.
2. (Trick) Construct dual spacetime metric and use geodesic principle.

Global Parameters

Integration constants for (N, ϕ) in SD Hamiltonian?

Interpretation

- Integration constants $\Rightarrow$ affect Hamiltonian.
- Test matter fields 'feel' integration constants through minimal coupling to SD Hamiltonian.

$\therefore$ integration constants parametrize propagation of test fields in SD.

Methodology

(At least) 2 perspectives:

1. (Rigorous) Minimally couple matter to SD, take zero back-reaction limit, analyse propagation of fields.
2. (Trick) Construct dual spacetime metric and use geodesic principle.

$\Rightarrow$ pick option 2!

A Shape Dynamics Solution

Assume *static* (N, N^a, ϕ) (NOT most general sol'n) [1]

[1] Gomes 2014 and sg-Forbes (in preparation)

A Shape Dynamics Solution

Assume *static* (N, N^a, ϕ) (NOT most general sol'n) [1]

Dual Spacetime Metric

PDE's for (ϕ, N):

$$\nabla^2\Omega = 0 \qquad \nabla^2(\Omega N) = 0\,, \tag{2}$$

where $\Omega = \log\phi$.

[1] Gomes 2014 and sg-Forbes (in preparation)

A Shape Dynamics Solution

Assume *static* (N, N^a, ϕ) (NOT most general sol'n) [1]

Dual Spacetime Metric

PDE's for (ϕ, N):

$$\nabla^2\Omega = 0 \qquad \nabla^2(\Omega N) = 0\,, \tag{2}$$

where $\Omega = \log\phi$.

General Solution:

$$\Omega = a\,(1 + b/r) \qquad N\Omega = c\,(1 + d/r)\ . \tag{3}$$

[1] Gomes 2014 and sg-Forbes (in preparation)

A Shape Dynamics Solution

Assume *static* (N, N^a, ϕ) (NOT most general sol'n) [1]

Dual Spacetime Metric

PDE's for (ϕ, N):

$$\nabla^2\Omega = 0 \qquad \nabla^2(\Omega N) = 0\,, \tag{2}$$

where $\Omega = \log\phi$.

General Solution:

$$\Omega = a\,(1 + b/r) \qquad N\Omega = c\,(1 + d/r)\ . \tag{3}$$

Time units $\Rightarrow a = 1$, Spatial units $\Rightarrow c = 1$, Consistency $\Rightarrow d = -r$

[1] Gomes 2014 and sg-Forbes (in preparation)

A Shape Dynamics Solution

Assume *static* (N, N^a, ϕ) (NOT most general sol'n) [1]

Dual Spacetime Metric

PDE's for (ϕ, N):

$$\nabla^2\Omega = 0 \qquad\qquad \nabla^2(\Omega N) = 0\,, \tag{2}$$

where $\Omega = \log\phi$.

General Solution:

$$\Omega = a\,(1 + b/r) \qquad\qquad N\Omega = c\,(1 + d/r)\;. \tag{3}$$

Time units $\Rightarrow a = 1$, Spatial units $\Rightarrow c = 1$, Consistency $\Rightarrow d = -r$

$$\Rightarrow H_{\rm SD} = 2b - \tfrac{2b^2}{r}$$

[1] Gomes 2014 and sg-Forbes (in preparation)

A Shape Dynamics Solution

Assume *static* (N, N^a, ϕ) (NOT most general sol'n) [1]

Dual Spacetime Metric

PDE's for (ϕ, N):

$$\nabla^2\Omega = 0 \qquad\qquad \nabla^2(\Omega N) = 0\,, \tag{2}$$

where $\Omega = \log\phi$.

General Solution:

$$\Omega = a\,(1 + b/r) \qquad\qquad N\Omega = c\,(1 + d/r)\ . \tag{3}$$

Time units $\Rightarrow a = 1$, Spatial units $\Rightarrow c = 1$, Consistency $\Rightarrow d = -r$

$$\Rightarrow H_{\mathrm{SD}} = 2b - \frac{2b^2}{r}$$

Compare to ADM energy $\Rightarrow b = m/2$, where $m \equiv$ ADM energy.

[1] Gomes 2014 and sg-Forbes (in preparation)

Traversable Warmhole in SD?

A dual spacetime metric to the SD solution:

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right) \tag{4}$$

This is metric for an Einstein–Rosen bridge.

Traversable Warmhole in SD?

A dual spacetime metric to the SD solution:

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right) \tag{4}$$

This is metric for an Einstein–Rosen bridge.

Traversable Warmhole?

In spacetime picture:

- $\sqrt{-g} = 0$ when $r = m/2$ (horizon).

Traversable Warmhole in SD?

A dual spacetime metric to the SD solution:

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right) \tag{4}$$

This is metric for an Einstein–Rosen bridge.

Traversable Warmhole?

In spacetime picture:

- $\sqrt{-g} = 0$ when $r = m/2$ (horizon).
- $G_{00}(r = m/2) \propto \delta(r - m/2)$.

Traversable Warmhole in SD?

A dual spacetime metric to the SD solution:

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right) \tag{4}$$

This is metric for an Einstein–Rosen bridge.

Traversable Warmhole?

In spacetime picture:

- $\sqrt{-g} = 0$ when $r = m/2$ (horizon).
- $G_{00}(r = m/2) \propto \delta(r - m/2)$.
- Sourced by "exotic" matter in spacetime picture.

Traversable Warmhole in SD?

A dual spacetime metric to the SD solution:

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right) \tag{4}$$

This is metric for an Einstein–Rosen bridge.

Traversable Warmhole?

In spacetime picture:

- $\sqrt{-g} = 0$ when $r = m/2$ (horizon).
- $G_{00}(r = m/2) \propto \delta(r - m/2)$.
- Sourced by "exotic" matter in spacetime picture.

But, shape dynamics solution and parameters are all finite!

Traversable Warmhole in SD?

A dual spacetime metric to the SD solution:

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right) \tag{4}$$

This is metric for an Einstein–Rosen bridge.

Traversable Warmhole?

In spacetime picture:

- $\sqrt{-g} = 0$ when $r = m/2$ (horizon).
- $G_{00}(r = m/2) \propto \delta(r - m/2)$.
- Sourced by "exotic" matter in spacetime picture.

But, shape dynamics solution and parameters are all finite!

$\Rightarrow$ 'exotic' nature of matter is conformal.

Traversable Warmhole in SD?

A dual spacetime metric to the SD solution:

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right) \tag{4}$$

This is metric for an Einstein–Rosen bridge.

Traversable Warmhole?

In spacetime picture:

- $\sqrt{-g} = 0$ when $r = m/2$ (horizon).
- $G_{00}(r = m/2) \propto \delta(r - m/2)$.
- Sourced by "exotic" matter in spacetime picture.

But, shape dynamics solution and parameters are all finite!

$\Rightarrow$ 'exotic' nature of matter is conformal.

$\therefore$ SD ontology suggests that the Einstein–Rosen bridge is physical!

Quantum Consequences (??)

Possible Quantum Consequences I

2 Scenarios: (Standard QFT)

Possible Quantum Consequences I

2 Scenarios: (Standard QFT)

1. Standard

Non-Gaussian (and highly non-local) UV-Fixed Point:

$$\mathrm{Conf}(3,1) \sim \mathrm{SO}(4,2) \xrightarrow{\text{(soft) breaking}} \mathrm{ISO}(3,1)$$

Possible Quantum Consequences I

2 Scenarios: (Standard QFT)

1. Standard

Non-Gaussian (and highly non-local) UV-Fixed Point:

$$\mathrm{Conf}(3,1) \sim \mathrm{SO}(4,2) \xrightarrow{\text{(soft) breaking}} \mathrm{ISO}(3,1)$$

Shape Dynamics: $\mathrm{Conf}(3) \sim \mathrm{SO}(4,1) \in \mathrm{SO}(4,2)$ is a *residual* "finger print" of UV.

Possible Quantum Consequences I

2 Scenarios: (Standard QFT)

1. Standard

Non-Gaussian (and highly non-local) UV-Fixed Point:

$$\mathrm{Conf}(3,1) \sim \mathrm{SO}(4,2) \xrightarrow{\text{(soft) breaking}} \mathrm{ISO}(3,1)$$

Shape Dynamics: $\mathrm{Conf}(3) \sim \mathrm{SO}(4,1) \in \mathrm{SO}(4,2)$ is a *residual* "finger print" of UV.

$\Rightarrow$ non-local $\mathrm{Conf}(3)$ could help to characterise non-local $\mathrm{Conf}(3,1)$?!

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Relevant deformation (UV): $(\text{Cotton})^2$

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Relevant deformation (UV): $(\text{Cotton})^2$

$\Rightarrow$ anisotropic scaling ($+$ detailed balance)

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Relevant deformation (UV): $(\text{Cotton})^2$

$\Rightarrow$ anisotropic scaling (+ detailed balance)

$\therefore$ Power Counting Renormalizable!

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Relevant deformation (UV): $(\text{Cotton})^2$

$\Rightarrow$ anisotropic scaling ($+$ detailed balance)

$\therefore$ Power Counting Renormalizable!

New Aspirations:

- New theory space and field content.

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Relevant deformation (UV): $(\text{Cotton})^2$

$\Rightarrow$ anisotropic scaling (+ detailed balance)

$\therefore$ Power Counting Renormalizable!

New Aspirations:

- New theory space and field content.
- RG-Flow between local UV and non-local IR fixed points.

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Relevant deformation (UV): $(\text{Cotton})^2$

$\Rightarrow$ anisotropic scaling ($+$ detailed balance)

$\therefore$ Power Counting Renormalizable!

New Aspirations:

- New theory space and field content.
- RG-Flow between local UV and non-local IR fixed points.
- Conformal invariance is maintained throughout RG-flow.

Possible Quantum Consequences II

2. Non-Standard ('Hořava–Lifshitz-like')

Symmetry: Foliation-preserving conformal diffeomorphisms.

UV-Fixed Point is *local* Conf(3)

Relevant deformation (UV): $(\text{Cotton})^2$

$\Rightarrow$ anisotropic scaling ($+$ detailed balance)

$\therefore$ Power Counting Renormalizable!

New Aspirations:

- New theory space and field content.
- RG-Flow between local UV and non-local IR fixed points.
- Conformal invariance is maintained throughout RG-flow.
- No conformal anomaly in 3d.

Summary / Conclusions

- There is a foliation of ADM where the gauge-invariant degrees of freedom are conformally invariant.

Summary / Conclusions

- There is a foliation of ADM where the gauge-invariant degrees of freedom are conformally invariant.
- This suggests an ontological shift to Shape Dynamics motivated by:
  - The fact that only ratios of lengths are observable.

Summary / Conclusions

- There is a foliation of ADM where the gauge-invariant degrees of freedom are conformally invariant.
- This suggests an ontological shift to Shape Dynamics motivated by:
  - The fact that only ratios of lengths are observable.
  - Fixed points are conformal.

Summary / Conclusions

- There is a foliation of ADM where the gauge-invariant degrees of freedom are conformally invariant.
- This suggests an ontological shift to Shape Dynamics motivated by:
  - The fact that only ratios of lengths are observable.
  - Fixed points are conformal.
- Classical: global differences suggest new physics $\Rightarrow$ traversable warmhole.

Summary / Conclusions

- There is a foliation of ADM where the gauge-invariant degrees of freedom are conformally invariant.
- This suggests an ontological shift to Shape Dynamics motivated by:
  - The fact that only ratios of lengths are observable.
  - Fixed points are conformal.
- Classical: global differences suggest new physics $\Rightarrow$ traversable warmhole.
- Semi-Classical: Information loss in SD black holes?

Summary / Conclusions

- There is a foliation of ADM where the gauge-invariant degrees of freedom are conformally invariant.
- This suggests an ontological shift to Shape Dynamics motivated by:
  - The fact that only ratios of lengths are observable.
  - Fixed points are conformal.
- Classical: global differences suggest new physics $\Rightarrow$ traversable warmhole.
- Semi-Classical: Information loss in SD black holes?
- Quantum: new proposal for UV fixed point.

Thank You!