Rethinking microscopic causality via **noncommutative geometry**

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10th May 2015

Krajowy Naukowy Ośrodek Wiodący

Outline

1. Some comments on causality
2. Noncommutative geometry
3. Summary + an outlook into quantum gravity

Causality in relativity theory

- Special Relativity

Digression:

- The formalism of SR allows for 'tachions'. *[G. Feinberg, Phys. Rev.* **159** *1085 (1967)]*
- n-particle state (vacuum in particular) is not Lorentz invariant.

- General Relativity

- Chronology protection conjecture [Hawking]: causality, strong causality, stable causality, global hyperbolicity.

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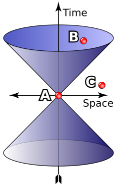

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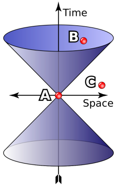

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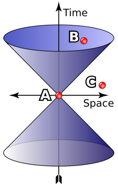

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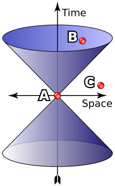

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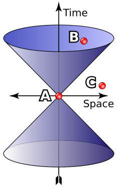

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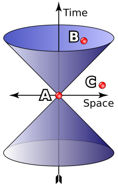

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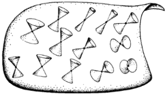

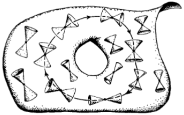

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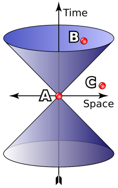

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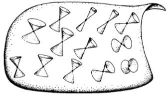

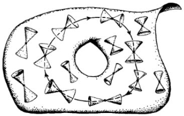

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Causality in quantum theory

- Causality in quantum mechanics
  - Initial controversies surrounding entangled states.
  - QI theorems: no-cloning, no-signalling, information causality, . . .
  - Non-locality $\neq$ causality violation!
- Causality in quantum field theory
  - Dirac and Klein–Gordon propagators vanish outside the light cone.
  - Microscopic causality [*Wightman, Kastler–Haag*]:

    "Observables with space-like separated supports commute."
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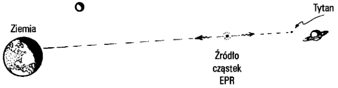

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Causality from mathematician's viewpoint

Causality – a partial order relation between points of space-time (events).

$$p \preceq q \iff \exists \text{ future directed causal curve } \gamma \text{ from } p \text{ to } q \text{ (or } p = q).$$

Properties:

- Induced by a Lorentzian metric on a manifold.
- Possible only in Lorentzian signature!
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Basic idea

locally compact Hausdorff topological spaces	$\xleftrightarrow{\text{Gelfand–Naimark thm}}$	commutative C^*-algebras
(pseudo)-Riemannian smooth manifolds	$\xleftrightarrow{\text{Connes' Reconstruction Thm}}$	commutative spectral triples
noncommutative space(-times)	$\xleftarrow{:=}$	noncommutative spectral triples

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Topology $\longleftrightarrow$ C^*-algebras

C^*-algebras

$(\mathcal{A}, +, \cdot)$ is an algebra with

- an involution $\forall\, a \in \mathcal{A} \quad \exists\, a^* \in \mathcal{A}$,
- a complete norm $\|\cdot\| : \mathcal{A} \to \mathbb{R}_+$,
- the $\mathbf{C}^*$**-property**: $\|aa^*\| = \|a\|^2$ (in general we only have $\|aa^*\| \leq \|a\|^2$).

Commutative Gelfand – Naimark theorem

commutative C^*-algbras $\overset{1:1}{\longleftrightarrow}$ (locally) compact Hausdorff topological spaces

Noncommutative Gelfand – Naimark theorem [1943]

noncommutative C^*-algbras $\overset{1:1}{\longleftrightarrow}$ bounded operators on a Hilbert space

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Geometry $\longleftrightarrow$ Dirac operators

- Consider a smooth (compact) spin Riemannian manifold M.
- Take $\mathcal{H} = L^2(M, S)$ and $\not{D} = -i\gamma^\mu \nabla^S_\mu = -i\gamma^\mu(\partial_\mu + \omega_\mu)$.

What is $\not{D}$ good for?

1. Its spectrum encodes the dimension of M.

$$\#\{\lambda(|\not{D}|) < \Lambda\} \sim c_n \cdot \mathrm{Vol}(M)\Lambda^n, \qquad \text{as } \Lambda \to \infty, \qquad \text{with } n = \dim M$$

2. It gives you differential forms: $f[\not{D}, g],\ f, g, \in C(M)$.
3. It tells you how to integrate functions [Connes' Trace Thm].

$$⨍ f|\not{D}|^{-n} := \operatorname*{Res}_{s=n} \mathrm{Tr}\, f|\not{D}|^{-s} = C_n \int_M f(x) d\mu_g(x), \qquad \forall f \in C(M).$$

4. It provides the geodesic distance.

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Spectral triples

$(\mathcal{A}, \mathcal{H}, \mathcal{D})$ – spectral triple

- $\mathcal{A}$ – pre-C^*-algebra (unital)
- $\mathcal{H}$ – Hilbert space, $\exists$ a faithful representation $\pi(\mathcal{A}) \subset \mathcal{B}(\mathcal{H})$
- $\mathcal{D}$ – the Dirac operator – selfadjoint, unbounded
  - $|\mathcal{D}|^{-1}$ – is a compact operator
  - $[\mathcal{D}, \pi(a)] \in \mathcal{B}(\mathcal{H})$ for all $a \in \mathcal{A}$

In the Lorentzian setting: need for indefinite product.

Hilbert space	$\rightsquigarrow$	Krein space
$\langle \cdot, \cdot \rangle$	$\rightsquigarrow$	$(\cdot, \cdot) = \langle \cdot, \mathfrak{J} \cdot \rangle$

- $\mathfrak{J}$ – fundamental symmetry operator – captures the signature.
- For a Lornetzian manifold $\mathfrak{J} = \gamma^0 \Rightarrow \overline{\psi} = \psi^\dagger \gamma^0$.

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- $\mathcal{A}$ – pre-C^*-algebra (unital)
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In the Lorentzian setting: need for indefinite product.

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Algebraisation of causality

$$p \preceq q \iff \exists \text{ future directed causal curve } \gamma \text{ from } p \text{ to } q \text{ (or } p = q).$$

Algebraisation:

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Theorem [N. Franco, M.E. (2013)]

M – globally hyperbolic space-time, $(\mathcal{A}_M, \mathcal{H}_M, \not{D})$ – spectral triple associated (canonically) with M. Then,

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Almost commutative space-times

- Finite spectral triples:

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- $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{C}$,
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Theorem [*N. Franco, M.E. 2015b*]

$(p,0) \preceq (q,1)$ iff $p \preceq q$ in M and

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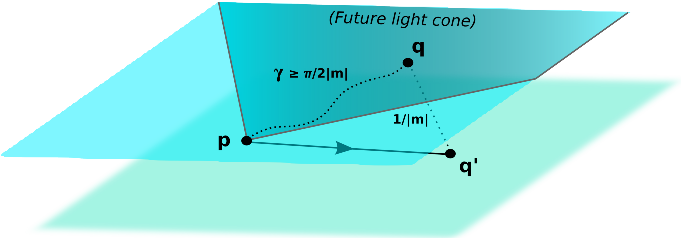

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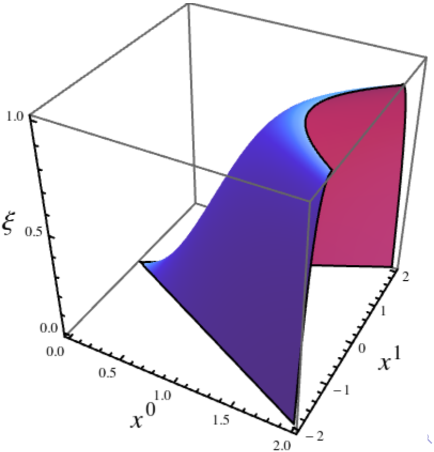

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- Zitterbewegung – the ‘trembling motion of the electron’.
- Free Dirac equation $i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$.
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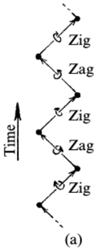

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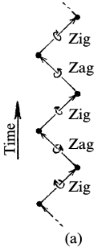

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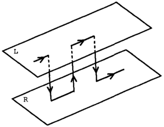

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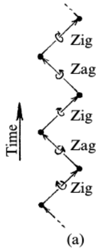

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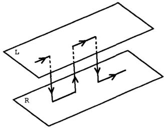

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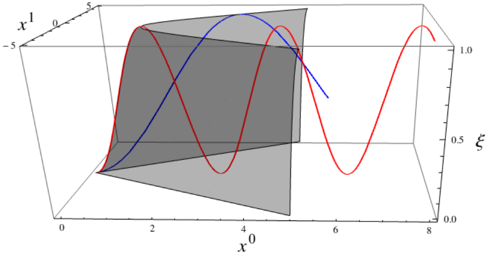

[M.E., N. Franco, T. Miller (2015c)]

Zitterbewegung of *interacting* fermions

- Fluctuations of Dirac operator: $\mathcal{D} \rightsquigarrow \mathcal{D}_{\mathbb{A}} = (\not{D} - \not{A}) \otimes 1 + \gamma_M \otimes \Phi$.
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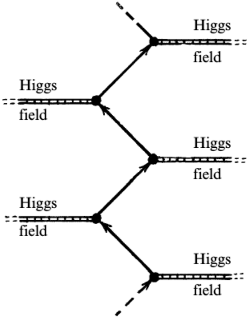

[R. Penrose, *Road to Reality*, 2004]

Zitterbewegung of *interacting* fermions

- Fluctuations of Dirac operator: $\mathcal{D} \rightsquigarrow \mathcal{D}_{\mathbb{A}} = (\not{D} - \not{A}) \otimes 1 + \gamma_M \otimes \Phi$.
- $S_F = \int_M d\mu \left(\overline{\psi}\not{D}\psi - \overline{\psi}\not{A}\psi - \Phi\overline{\psi}\psi\right)$

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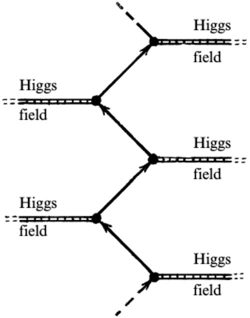

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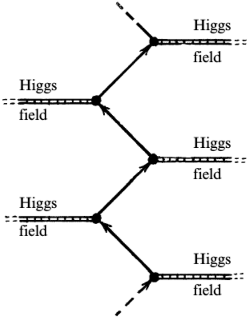

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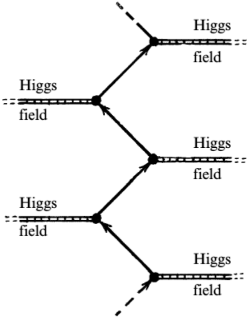

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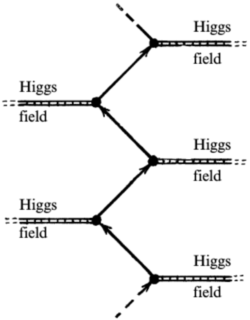

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Outline

Take-home messages

- Noncommutative geometry offers a robust framework for physical theories.
- Matter does matter! $\longrightarrow$ It refines microscopic causality.
- Spectral triples for QFT? [Besnard, Rovelli, Verch]
- Spectral triples for: fractals, Loop Quantum Gravity, quantum deformations,
- The concept of causality in the space of states is universal.
- Space-time is not a fundamental concept, events are not observables!
  - Inherent non-locality in QFT.
  - Too precise measurements create event horizons [DFR (1994)].

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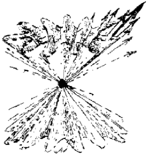

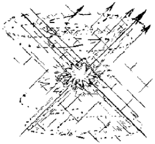

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