The κ-Carroll particle from 3d gravity

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Outline:

1. A point particle coupled to 3d gravity
   - 3d gravity as the Chern-Simons theory
   - Gravitating particle in flat spacetime

2. A deformed Carroll particle

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1. A point particle coupled to 3d gravity
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Motivation

- There is a variety of problems in the search for quantum gravity
- 3d gravity is a simpler version of ordinary general relativity
- It has no local degrees of freedom i.e. no gravitational waves
- Dynamics can be reintroduced via a nontrivial spacetime topology
- The 3d Newton's constant has the dimension of inverse mass
- Momentum space of a point particle coupled to 3d gravity is curved
- Curved momentum space is characteristic to some of the approaches to quantum gravity, especially "relative locality"

De Sitter gauge group

For the cosmological constant $\Lambda > 0$ the local isometry group of space-time is de Sitter group $\mathrm{SO}(3,1)$. Its algebra has the commutators

$$[J_\mu, J_\nu] = \epsilon_{\mu\nu\sigma} J^\sigma \,, \qquad [J_\mu, P_\nu] = \epsilon_{\mu\nu\sigma} P^\sigma \,,$$
$$[P_\mu, P_\nu] = -\Lambda\, \epsilon_{\mu\nu\sigma} J^\sigma \,, \tag{1}$$

where $\mu, \nu, \sigma = 0, 1, 2$. Introducing new generators $S_\mu \equiv P_\mu + \sqrt{\Lambda}\, \epsilon_{\mu 0 \nu} J^\nu$ we may rewrite it as

$$[J_\mu, J_\nu] = \epsilon_{\mu\nu\sigma} J^\sigma \,, \qquad [J_\mu, S_\nu] = \epsilon_{\mu\nu\sigma} S^\sigma + \sqrt{\Lambda}\, (\eta_{\nu 0} J_\mu - \eta_{\mu\nu} J_0) \,,$$
$$[S_\mu, S_\nu] = \sqrt{\Lambda}\, (\eta_{\mu 0} S_\nu - \eta_{\nu 0} S_\mu) \,. \tag{2}$$

Thus group elements $\gamma \in \mathrm{SO}(3,1)$ can be locally factorized into

$$\gamma = \mathfrak{j}\, \mathfrak{s} = (\iota_3 + \iota^\mu J_\mu)(\xi_3 + \xi^\nu S_\nu) \,, \tag{3}$$

with $\mathfrak{j} \in \mathrm{SO}(2,1)$, $\mathfrak{s} \in \mathrm{AN}(2)$ and $\iota_3^2 + \frac{1}{4} \iota_\mu \iota^\mu = 1$, $\xi_3^2 - \frac{\Lambda}{4} \xi_0^2 = 1$. We also have the natural scalar product

$$\langle J_\mu S_\nu \rangle = \eta_{\mu\nu} \,, \qquad \langle J_\mu J_\nu \rangle = \langle S_\mu S_\nu \rangle = 0 \,. \tag{4}$$

Chern-Simons action of 3d gravity

Instead of the metric $g_{\alpha\beta}$ gravity may be described by the vielbein $e_\alpha^{\ \mu}$ and spin connection $\omega_\alpha^{\ \mu\nu}$, defined through

$$e_\alpha^{\ \mu} e_\beta^{\ \nu} \eta_{\mu\nu} = g_{\alpha\beta}\,, \qquad \omega_\alpha^{\ \mu\nu} = e_\beta^{\ \mu} \partial_\alpha e^{\beta\nu} + e_\beta^{\ \mu} \Gamma^\beta_{\ \alpha\gamma} e^{\gamma\nu}\,. \tag{5}$$

For 3d gravity we can introduce the gauge field, which is the Cartan connection

$$A = \omega^\mu J_\mu + e^\mu P_\mu\,, \tag{6}$$

where $e^\mu = e_\alpha^{\ \mu} dx^\alpha$ and $\omega^\mu = -\frac{1}{2}\epsilon^\mu_{\ \nu\sigma}\omega_\alpha^{\ \nu\sigma} dx^\alpha$, and the Einstein-Hilbert action can be written as the Chern-Simons gauge theory

$$S = \frac{k}{4\pi} \int \left(\langle dA \wedge A \rangle + \frac{1}{3} \langle A \wedge [A, A] \rangle \right)\,, \tag{7}$$

with the coupling constant $k \equiv \frac{1}{4G}$.

Coupling of a particle to 3d gravity

If we separate time $\mathbb{R}$ and space $\mathcal{S}$ then A may be split into $A = A_t dt + A_S$. The action of gravity with a point particle is given by

$$S = \int dt\, L = \frac{k}{4\pi} \int dt \int_{\mathcal{S}} \left\langle \dot{A}_S \wedge A_S \right\rangle - \int dt \left\langle \mathcal{C}\, h^{-1} \dot{h} \right\rangle + \int dt \int_{\mathcal{S}} \left\langle A_t \left(\frac{k}{2\pi} F_S - h \mathcal{C}\, h^{-1} \delta^2(\vec{y})\, dy^1 \wedge dy^2 \right) \right\rangle . \tag{8}$$

The algebra element $\mathcal{C} = m\, J_0 + s\, S_0$ encodes the particle's mass m and spin s and the group element h describes the particle's motion. The spatial curvature $F_S = dA_S + [A_S, A_S]$ satisfies the constraint

$$\frac{k}{2\pi} F_S = h \mathcal{C}\, h^{-1} \delta^2(\vec{y})\, dy^1 \wedge dy^2 \,, \tag{9}$$

i.e. vanishes everywhere except a singularity at the particle's worldline.

Alekseev-Malkin construction

Space $\mathcal{S}$ may be decomposed into the disc $\mathcal{D}$ surrounding the particle, with coordinates $r \in [0,1]$, $\phi \in [0,2\pi]$, and the asymptotic empty region $\mathcal{E}$ (for $r \geq 1$), with the common boundary Γ at $r = 1$. Solving the curvature constraint we find that on $\mathcal{E}$ the connection has the form

$$A_S^{(\mathcal{E})} = \gamma d\gamma^{-1}\,, \tag{10}$$

while on $\mathcal{D}$ it is given by

$$A_S^{(\mathcal{D})} = \bar{\gamma}\,\frac{1}{k}\mathcal{C}d\phi\,\bar{\gamma}^{-1} + \bar{\gamma}d\bar{\gamma}^{-1}\,, \quad \bar{\gamma}(r=0) = h\,, \tag{11}$$

where γ, $\bar{\gamma}$ are some gauge group elements. The continuity of A_S across Γ, i.e. $A_S^{(\mathcal{D})}|_\Gamma = A_S^{(\mathcal{E})}|_\Gamma$ leads to the sewing condition

$$\gamma^{-1}|_\Gamma = N\,e^{\frac{1}{k}\mathcal{C}\phi}\bar{\gamma}^{-1}|_\Gamma\,, \tag{12}$$

where $N = N(t)$ is an arbitrary gauge group element.

Poincaré gauge group

In the limit $\Lambda \to 0$ the group $\mathrm{SO}(3,1)$ becomes the Poincaré group $\mathrm{ISO}(2,1) \simeq \mathfrak{so}(2,1)^* \rtimes \mathrm{SO}(2,1)$, $\mathfrak{so}(2,1)^* \simeq \mathbb{R}^3$, with the algebra

$$[J_\mu, J_\nu] = \epsilon_{\mu\nu\sigma} J^\sigma \,, \qquad [J_\mu, P_\nu] = \epsilon_{\mu\nu\sigma} P^\sigma \,, \qquad [P_\mu, P_\nu] = 0 \,. \tag{13}$$

Group elements $\gamma \in \mathrm{ISO}(2,1)$ are factorized into

$$\gamma = \mathfrak{j}\,\mathfrak{p} = (\iota_3 + \iota^\mu J_\mu)(1 + \xi^\nu P_\nu)\,, \tag{14}$$

with $\mathfrak{j} \in \mathrm{SO}(2,1)$, $\xi \equiv \xi^a P_a \in \mathfrak{so}(2,1)^*$, and the group multiplication

$$\gamma_{(1)}\gamma_{(2)} = \mathfrak{j}_{(1)}\mathfrak{j}_{(2)} \left(1 + \mathrm{Ad}(\mathfrak{j}_{(2)}^{-1})\,\xi_{(1)} + \xi_{(2)}\right) . \tag{15}$$

The scalar product again has the form

$$\langle J_\mu P_\nu \rangle = \eta_{\mu\nu} \,, \qquad \langle J_\mu J_\nu \rangle = \langle P_\mu P_\nu \rangle = 0 \,. \tag{16}$$

Effective particle Lagrangian

We can now put the Lagrangian in the boundary form

$$L = \frac{k}{2\pi}\int_\Gamma \left\langle \mathfrak{j}^{-1}\dot{\mathfrak{j}}\, d\xi - \bar{\mathfrak{j}}^{-1}\dot{\bar{\mathfrak{j}}}\, d\bar{\xi} + \frac{\mathcal{C}_J}{k} d\phi \left[\bar{\mathfrak{j}}^{-1}\dot{\bar{\mathfrak{j}}}, \bar{\xi}\right] + \frac{\mathcal{C}_P}{k} d\phi\, \bar{\mathfrak{j}}^{-1}\dot{\bar{\mathfrak{j}}} \right\rangle , \quad (17)$$

where $\mathcal{C}_J \equiv m\, J_0$, $\mathcal{C}_P \equiv s\, P_0 = s\, S_0$. The sewing condition splits into

$$\mathfrak{j}^{-1} = \mathfrak{n}\, e^{\frac{1}{k}\mathcal{C}_J \phi} \bar{\mathfrak{j}}^{-1} , \qquad -\mathrm{Ad}(\mathfrak{n}^{-1})\, \xi = \nu - \mathrm{Ad}(e^{\frac{1}{k}\mathcal{C}_J \phi})\, \bar{\xi} + \tfrac{1}{k}\mathcal{C}_P \phi , \quad (18)$$

where $N = (1+\nu)\,\mathfrak{n}$, $\mathfrak{n} \in \mathrm{SO}(2,1)$, $\nu \in \mathfrak{so}(2,1)^*$. Substituting these conditions and denoting $\kappa \equiv \frac{k}{2\pi}$ we eventually obtain the Lagrangian

$$L = \kappa \left(\dot{\Pi}^{-1}\Pi\right)_\mu x^\mu + s \left(\mathfrak{n}^{-1}\dot{\mathfrak{n}}\right)_0 , \quad (19)$$

with the particle's position $x \equiv \mathfrak{n}\, \bar{\xi}\, \mathfrak{n}^{-1} \in \mathfrak{so}(2,1)^*$ and momentum $\Pi \equiv \mathfrak{n}\, e^{\frac{m}{\kappa} J_0} \mathfrak{n}^{-1} \in \mathrm{SO}(2,1)$.

Properties of the gravitating particle

The parallel transport around the particle is described by the holonomy of the connection A_S along the boundary Γ, which is given by

$$\begin{aligned}\mathrm{hol}_\Gamma(A_S) = \gamma(\phi = 0)\,\gamma^{-1}(\phi = 2\pi) = \\ \Pi\left(1 + \left(\mathrm{Ad}(\Pi^{-1}) - 1\right)x + \mathrm{Ad}(\Pi^{-1})\,\bar{\mathrm{j}}\,\tfrac{1}{\kappa}\mathcal{C}_P\bar{\mathrm{j}}^{-1}\right) .\end{aligned} \tag{20}$$

In particular, $\mathrm{j}(\phi = 0)\,\mathrm{j}^{-1}(\phi = 2\pi) = \Pi$. The momentum manifold $\mathrm{SO}(2,1)$ is 3d anti-de Sitter space. Using the parametrization $\Pi = p_3 + \frac{1}{\kappa}\,p^\mu J_\mu$, where $p_3 = \sqrt{1 - \frac{1}{4\kappa^2}p_\mu p^\mu}$, we find the mass shell condition

$$p_\mu p^\mu = 4\kappa^2 \sin^2 \frac{m}{2\kappa} . \tag{21}$$

Properties of the gravitating particle – cont.

Let us restrict to the spinless case. With $x = x^\mu P_\mu$ and the Lagrange multiplier λ we may rewrite the effective Lagrangian as

$$L = -\left(p_3 \dot{p}_\mu - \dot{p}_3 p_\mu - \frac{1}{2\kappa}\,\epsilon_{\mu\nu\sigma}\dot{p}^\nu p^\sigma\right) x^\mu - \lambda\left(p_\mu p^\mu - 4\kappa^2 \sin^2 \frac{m}{2\kappa}\right) . \tag{22}$$

It still leads to the equations of motion of a free relativistic particle

$$\dot{x}^\mu = 2\lambda\, p^\mu \,, \qquad \dot{p}_\mu = 0 \tag{23}$$

and in the limit $\kappa \to \infty$ we recover the free particle Lagrangian

$$L = -\dot{p}_\mu x^\mu - \lambda\left(p_\mu p^\mu - m^2\right) . \tag{24}$$

The case of multiple particles

The Chern-Simons Lagrangian for a system of n particles has the form

$$L_{(n)} = \frac{k}{4\pi} \int_{\mathcal{S}} \left\langle \dot{A}_S \wedge A_S \right\rangle - \sum_{i=1}^{n} \left\langle \mathcal{C}_i h_i^{-1} \dot{h}_i \right\rangle +$$

$$\int_{\mathcal{S}} \left\langle A_t \left(\frac{k}{2\pi} F_S - \sum_{i=1}^{n} h_i \mathcal{C}_i h_i^{-1} \delta^2(\vec{y} - \vec{y}_i)\, dy^1 \wedge dy^2 \right) \right\rangle . \tag{25}$$

We may divide $\mathcal{S}$ into n particle discs $\mathcal{D}_i$ and the empty polygon $\mathcal{E}$, whose edges Γ_i coincide with the boundaries of $\mathcal{D}_i$'s. On each $\mathcal{D}_i$ the connection is given by

$$A_S^{(\mathcal{D}_i)} = \bar{\gamma}_i \frac{1}{k} \mathcal{C}_i d\phi_i \bar{\gamma}_i^{-1} + \bar{\gamma}_i d\bar{\gamma}_i^{-1} , \quad \bar{\gamma}_i(r_i = 0) = h_i . \tag{26}$$

Then for the i'th particle we can derive

$$L_i = \kappa \left(\dot{\Pi}_i^{-1} \Pi_i \right)_\mu x_i^\mu + s_i \left(\mathfrak{n}_i^{-1} \dot{\mathfrak{n}}_i \right)_0 . \tag{27}$$

The case of multiple particles – cont.

Imposing the continuity at the polygon's vertices, $\gamma(\phi_{i+1} = 0) = \gamma(\phi_i = 2\pi)$ and fixing $\gamma(\phi_1 = 0) = 1$ we obtain the sequence of conditions

$$\mathfrak{n}_1 \bar{\mathfrak{j}}_1^{-1} = 1\,, \qquad \mathfrak{n}_2 \bar{\mathfrak{j}}_2^{-1} = \bar{\Pi}_1\,, \qquad \mathfrak{n}_3 \bar{\mathfrak{j}}_3^{-1} = \bar{\Pi}_1 \bar{\Pi}_2\,, \qquad \ldots\,, \tag{28}$$

where $\bar{\Pi}_i = \bar{\mathfrak{j}}_i e^{\frac{1}{\kappa} m_i J_0} \bar{\mathfrak{j}}_i^{-1}$. Using them we find the n-particle Lagrangian

$$L_{(n)} = \sum_{i=1}^{n} \left(\kappa \left(\dot{\bar{\Pi}}_i^{-1} \bar{\Pi}_i \right)_\mu \bar{x}_i^\mu + s_i \left(\bar{\mathfrak{j}}_i^{-1} \dot{\bar{\mathfrak{j}}}_i \right)_0 \right) +$$
$$\kappa \left(\bar{\Pi}_2^{-1} \dot{\bar{\Pi}}_1^{-1} \bar{\Pi}_1 \bar{\Pi}_2 - \dot{\bar{\Pi}}_1^{-1} \bar{\Pi}_1 \right)_\mu \bar{x}_2^\mu + s_2 \left(\bar{\mathfrak{j}}_2^{-1} \bar{\Pi}_1^{-1} \dot{\bar{\Pi}}_1 \bar{\mathfrak{j}}_2 \right)_0 + \ldots\,, \tag{29}$$

where $\bar{x}_i = \bar{\mathfrak{j}}_i \bar{\xi}_i \bar{\mathfrak{j}}_i^{-1}$.

Alternative contraction of $SO(3,1)$

Let us rescale the $\mathfrak{so}(3,1)$ generators to $\tilde{J}_\mu \equiv \sqrt{\Lambda}\, J_\mu$, $\tilde{P}_\mu \equiv 1/\sqrt{\Lambda}\, P_\mu$ and define $\tilde{S}_\mu \equiv \tilde{P}_\mu + \epsilon_{\mu 0\nu}\tilde{J}^\nu$. In the limit $\Lambda \to 0$ we obtain the algebra

$$\begin{aligned} [\tilde{J}_\mu, \tilde{J}_\nu] = 0\,, \qquad [\tilde{J}_\mu, \tilde{S}_\nu] = \eta_{\nu 0}\tilde{J}_\mu - \eta_{\mu\nu}\tilde{J}_0\,, \\ [\tilde{S}_\mu, \tilde{S}_\nu] = \eta_{\mu 0}\tilde{S}_\nu - \eta_{\nu 0}\tilde{S}_\mu\,. \end{aligned} \tag{30}$$

It generates the group $AN(2) \ltimes \mathfrak{an}(2)^*$, whose elements are

$$\gamma = \mathfrak{i}\,\mathfrak{s} = (1 + \iota^\mu \tilde{J}_\mu)(\xi_3 + \xi^\nu \tilde{S}_\nu)\,, \tag{31}$$

with $\mathfrak{s} \in \mathrm{AN}(2)$, $\iota \equiv \iota^\mu \tilde{J}_\mu \in \mathfrak{an}(2)^* \simeq \mathbb{R}^3$, and the group multiplication

$$\gamma_{(1)}\gamma_{(2)} = \left(1 + \iota_{(1)} + \mathrm{Ad}\left(\mathfrak{s}_{(1)}\right)\iota_{(2)}\right)\mathfrak{s}_{(1)}\mathfrak{s}_{(2)}\,. \tag{32}$$

The corresponding scalar product is

$$\left\langle \tilde{J}_\mu \tilde{S}_\nu \right\rangle = \eta_{\mu\nu}\,, \qquad \left\langle \tilde{J}_\mu \tilde{J}_\nu \right\rangle = \left\langle \tilde{S}_\mu \tilde{S}_\nu \right\rangle = 0\,. \tag{33}$$

New effective particle Lagrangian

We also have to exchange the labels of mass and spin so that $\mathcal{C}_{\tilde{J}} = s\,\tilde{J}_0$ and $\mathcal{C}_{\tilde{S}} = m\,\tilde{S}_0$. Then we put the Lagrangian in the boundary form

$$L = \frac{k}{2\pi}\int_\Gamma \left\langle \dot{\mathfrak{J}}\mathfrak{J}^{-1}\left(d\bar{\mathfrak{s}}\,\bar{\mathfrak{s}}^{-1} - \bar{\mathfrak{s}}\,\frac{1}{k}\mathcal{C}d\phi\,\bar{\mathfrak{s}}^{-1}\right) + \frac{1}{k}\mathcal{C}d\phi\,\bar{\mathfrak{s}}^{-1}\dot{\bar{\mathfrak{s}}}\right\rangle\,, \qquad (34)$$

where we denote $\mathfrak{J} \equiv \bar{\mathfrak{j}}^{-1}\mathfrak{j}$. The sewing condition can be split into

$$\mathfrak{s}^{-1} = \mathfrak{v}\,e^{\frac{1}{k}\mathcal{C}_{\tilde{S}}\phi}\bar{\mathfrak{s}}^{-1}\,, \qquad \mathfrak{J} = e^{-\frac{1}{k}\mathcal{C}_{\tilde{J}}\phi}\mathfrak{s}\,(1-n)\,\mathfrak{s}^{-1}\,, \qquad (35)$$

where $N = (1+n)\,\mathfrak{v}$, $\mathfrak{v} \in \mathrm{AN}(2)$, $n \in \mathfrak{an}(2)^*$. Substituting it we obtain the final Lagrangian

$$L = \kappa\left(\dot{\Pi}\,\Pi^{-1}\right)_\mu x^\mu + s\left(\bar{\mathfrak{s}}^{-1}\dot{\bar{\mathfrak{s}}}\right)_0\,, \qquad (36)$$

with the particle's momentum $\Pi \equiv \bar{\mathfrak{s}}\,e^{\frac{m}{\kappa}\tilde{S}_0}\bar{\mathfrak{s}}^{-1} \in \mathrm{AN}(2)$ and position $x \equiv \bar{\mathfrak{s}}\,\mathfrak{v}^{-1}n\,\mathfrak{v}\,\bar{\mathfrak{s}}^{-1} \in \mathfrak{an}(2)^*$.

Properties of the particle

The holonomy of the connection A_S along the boundary Γ is given by

$$\mathrm{hol}_\Gamma(A_S) = \gamma(\phi = 0)\,\gamma^{-1}(\phi = 2\pi) = \\ \left(1 + (1 - \mathrm{Ad}(\Pi))\,x + \tfrac{1}{\kappa}\mathcal{C}_{\tilde{J}}\right)\Pi\,. \tag{37}$$

In particular, $\mathfrak{s}(\phi = 0)\,\mathfrak{s}^{-1}(\phi = 2\pi) = \Pi$. The momentum manifold $\mathrm{AN}(2)$ is 3d de Sitter space. Furthermore, using the parametrization $\Pi = e^{p^a/\kappa\,\tilde{S}_a} e^{p^0/\kappa\,\tilde{S}_0}$, $\mathfrak{v} = e^{\upsilon^a/\kappa\,\tilde{S}_a} e^{\upsilon^0/\kappa\,\tilde{S}_0}$, $a = 1,2$ we find that

$$p^0 = m\,, \qquad p^a = \left(1 - e^{\frac{m}{\kappa}}\right)\upsilon^a\,, \tag{38}$$

i.e. the particle's energy is fixed to be the rest energy.

Properties of the particle – cont.

The spin term does not contribute to the equations of motion and we may restrict to $s = 0$. With $x = x^\mu \tilde{P}_\mu$ and the Lagrange multiplier λ the particle Lagrangian can be rewritten in the form

$$L = -\left(x^0 \dot{p}_0 + x^a \dot{p}_a - \kappa^{-1} x^a p_a \dot{p}_0\right) - \lambda\left(p_0^2 - m^2\right) , \tag{39}$$

which describes a κ-deformed Carroll particle. It gives the equations of motion of an ordinary Carroll particle

$$\dot{x}^0 = 2\lambda\, m\,, \qquad \dot{x}^a = 0\,, \qquad \dot{p}_\mu = 0 \tag{40}$$

and in the limit $\kappa \to \infty$ we recover the Carroll particle Lagrangian

$$L = -x^0 \dot{p}_0 - x^a \dot{p}_a - \lambda\left(p_0^2 - m^2\right) . \tag{41}$$

Symmetries of the deformed Carroll particle

The particle is invariant under infinitesimal κ-deformed Carroll transformations, which include ordinary rotations

$$\delta x^a = \rho\,\epsilon^a{}_b x^b\,, \qquad \delta p_a = \rho\,\epsilon_a{}^b p_b\,, \qquad \delta x^0 = \delta p_0 = 0\,, \tag{42}$$

deformed Carrollian boosts

$$\delta x^0 = \left(1 + \kappa^{-1} p_0\right) \lambda_a x^a\,, \qquad \delta p_a = -\lambda_a p_0\,, \qquad \delta x^a = \delta p_0 = 0\,, \tag{43}$$

deformed translations

$$\delta x^0 = \alpha^0\,, \qquad \delta x^a = e^{p_0/\kappa} \alpha^a\,, \qquad \delta p_\mu = 0 \tag{44}$$

and spatial conformal transformations

$$\delta x^a = \eta\, x^a\,, \qquad \delta p_a = -\eta\, p_a\,, \qquad \delta x^0 = \delta p_0 = 0\,, \tag{45}$$

where ρ, λ_a, α^μ, η denote transformation parameters.

Multiple particles

At the boundary Γ_i of each disc $\mathcal{D}_i$, $i = 1, \ldots, n$ we find

$$L = \kappa \left(\dot{\Pi}_i \, \Pi_i^{-1} \right)_\mu x_i^\mu + s_i \left(\bar{\mathfrak{s}}_i^{-1} \dot{\bar{\mathfrak{s}}}_i \right)_0 . \tag{46}$$

The continuity $\gamma(\phi_{i+1} = 0) = \gamma(\phi_i = 2\pi)$ and fixing $\gamma(\phi_1 = 0) = 1$ leads to the sequence of conditions

$$\mathfrak{v}_1 \bar{\mathfrak{s}}_1^{-1} = 1 , \qquad \mathfrak{v}_2 \bar{\mathfrak{s}}_2^{-1} = \bar{\Pi}_1 , \qquad \mathfrak{v}_3 \bar{\mathfrak{s}}_3^{-1} = \bar{\Pi}_2 \bar{\Pi}_1 , \qquad \ldots , \tag{47}$$

where $\bar{\Pi}_i = \mathfrak{v}_i e^{\frac{1}{\kappa} m_i \tilde{S}_0} \mathfrak{v}_i^{-1}$. Then we obtain the n-particle Lagrangian

$$\begin{aligned} L_{(n)} = \sum_{i=1}^{n} \left(\kappa \left(\dot{\bar{\Pi}}_i \bar{\Pi}_i^{-1} \right)_\mu \bar{x}_i^\mu + s_i \left(\mathfrak{v}_i^{-1} \dot{\mathfrak{v}}_i \right)_0 \right) + \\ \kappa \left(\bar{\Pi}_2 \dot{\bar{\Pi}}_1 \bar{\Pi}_1^{-1} \bar{\Pi}_2^{-1} - \dot{\bar{\Pi}}_1 \bar{\Pi}_1^{-1} \right)_\mu \bar{x}_2^\mu + s_2 \left(\mathfrak{v}_2^{-1} \bar{\Pi}_1 \dot{\bar{\Pi}}_1^{-1} \mathfrak{v}_2 \right)_0 + \ldots , \end{aligned} \tag{48}$$

where $\bar{x}_i = n_i$.

Summary

- We rederived the action of a gravitating particle in flat spacetime
- We obtained the new action of a κ-deformed Carroll particle
- Its momentum space is equivalent to the 3d κ-Minkowski momentum space, associated with the κ-Poincaré Hopf algebra
- The relevance of this particle model remains to be understood

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