RMT: Applications in the Information Era
April 30th 2019 Kraków

Equal-time and lagged correlations in human EEG

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Questions

(Some) Questions in Neuroscience

- Biomarkers
- Diagnosis
- Prediction
- Control

(Some) Questions in Neuroscience

- Diffrences between groups of people (clinical: healthy vs diseased/disordered)
- Differences in processing tasks (cognitive: memory, language, conflict, emotions, etc.)
- Where things happen?
- When things happen?
- In what order?
- What are the information pathways?

(Some) Physicists' Questions in Neuroscience

- Is it at a critical point?
- How (multi)stable it is?
- In what way it is self-organised?
- How does pattern formation appear there?
- … [your favourite question] …

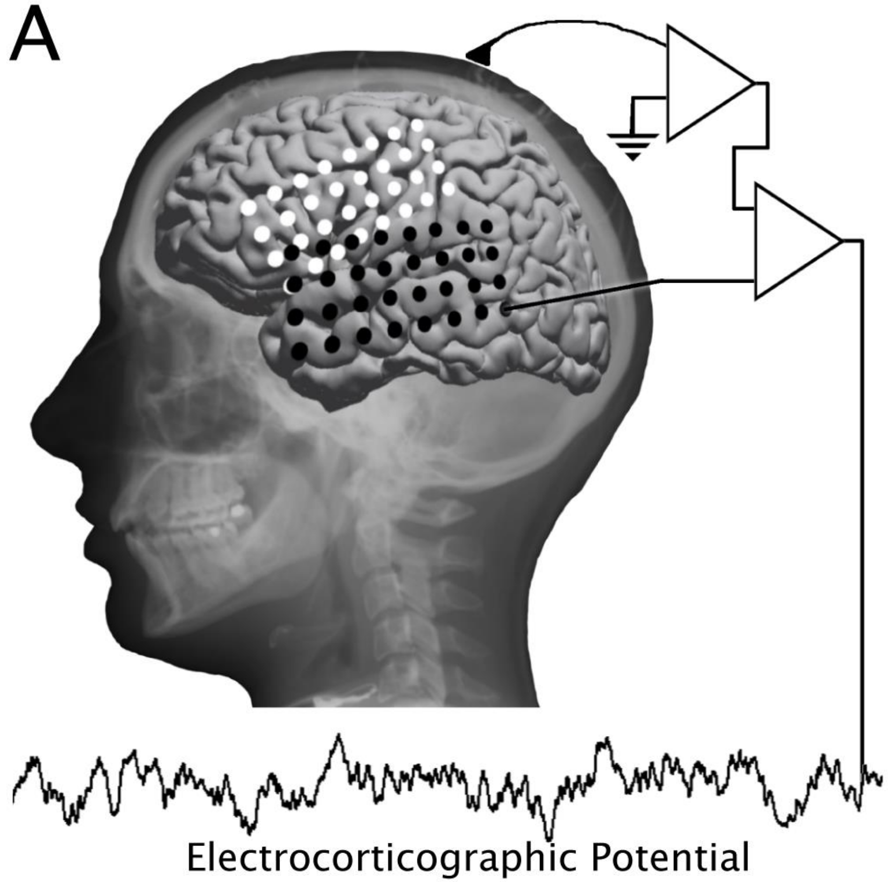
Types of measurement techniques

What is measured
Channels
Resolution

Measures

- Questionnaires
- Behavioural data
  - responses
  - reaction times
- (Neuro)physiological data:
  - breathing, heart rate, movement
  - hormone concentrations
  - neural activity
  - …
- Genetics

Electrocorticography (ECoG)



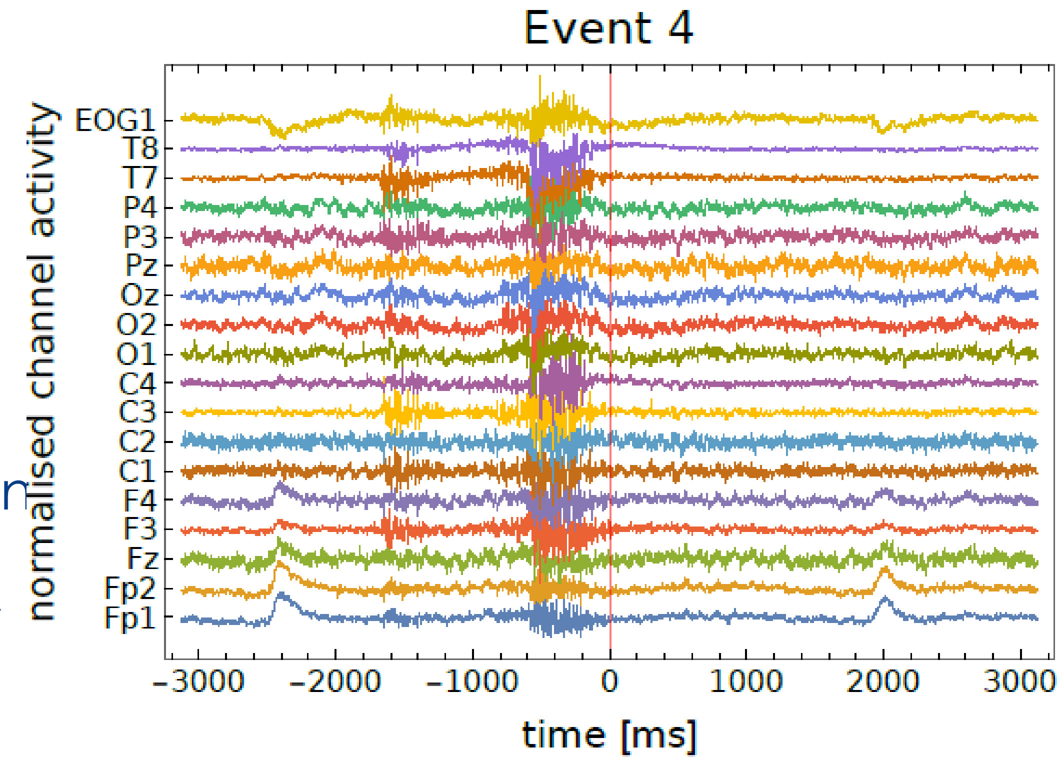
Miller KJ, Sorensen LB, Ojemann JG, den Nijs M (2009) Power-Law Scaling in the Brain Surface Electric Potential. PLoS Comput Biol 5(12): e1000609

Electroencephalography (EEG)

- scalp electrodes (non-invasive)
- mixed sources
- not easy to associate signals with anatomical sources

Electroencephalography (EEG)

- Time scales:
  - typical recording 30-60 minutes
  - Sampling ~500 Hz
  - human reaction >150 ms
  - Neural transmission ~10-20 ms
  - Neuron refractory period~1 ms
- Spatial scales:
  - 5-256 electrodes
  - Cortex surface 2,500 cm^2
  - ~20 million/cm^2 (human cortex)



Functional MRI (fMRI)

- blood-oxygen level–dependent (**BOLD**) signal
- Time scales:
  - TR~1-3 s
  - haemodynamic response~20 s
  - typical scan 5-15 minutes
- Spatial scales:
  - voxels~9 mm^2
  - brain~60x70x60
  - anatomical ROIs ~100-1000 vx
  - resting-state networks ~1000-10 000 vx

Functional MRI (fMRI)

- locations of signals known
- correlations are more interpretable
- strength and patterns of functional correlations vary across
  - cognitive states
  - conscious states
  - brain disorders
  - …

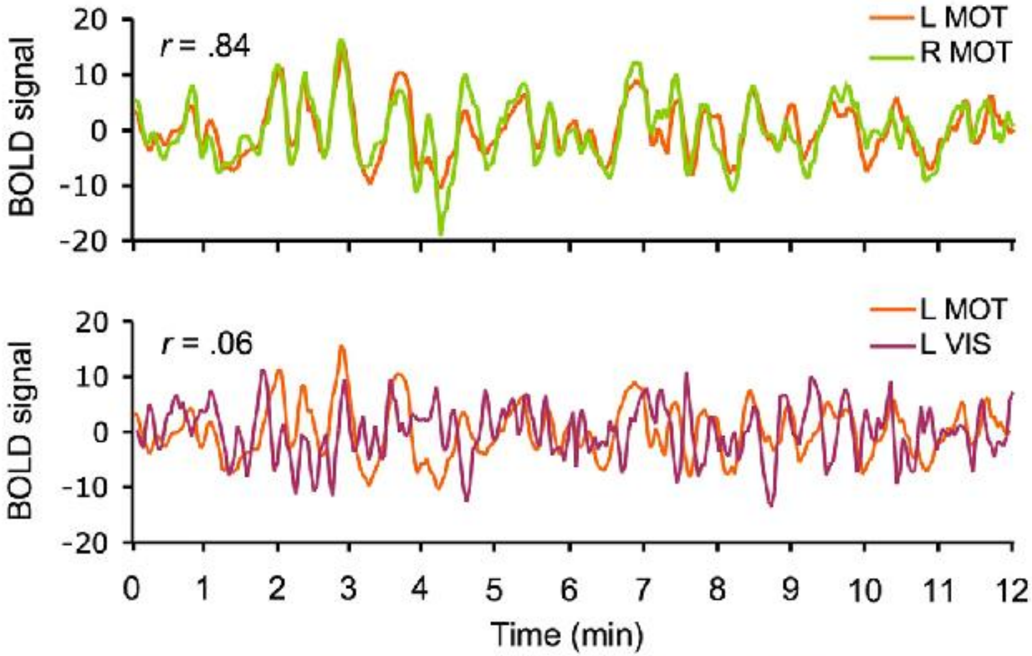


FIG. 1. The basis of functional connectivity MRI (fcMRI). Low-frequency spontaneous fluctuations in the blood-oxygenation-level-dependent (BOLD) signal are correlated over time between regions within the same brain systems.

Van Dijk KR, Hedden T, et al.Intrinsic functional connectivity as a tool for human connectomics: theory, properties, and optimization. J Neurophysiol 2010;103(1):297–321.

RMT applications in neuroscience

- If signals are modelled by iid Gaussian random variables, with $N, T \rightarrow \infty$, and constant rectangularity $r = N/T$, the eigenspectrum of Pearson correlation is well known.
- EEG naively should reduce noise much better (small *r*) than fMRI (very large *r*)

Matrices

- Phase synchronisation, coherence, average phase synchronisation time, etc.

$$R_{ij} = \left| \frac{1}{n} \sum_{l=1}^{n} \exp\left(\mathrm{i}\left(\phi_{jl} - \phi_{il}\right)\right) \right|$$

i, *j* – (oscillatory) channels, φ_i – instantaneous phase, *l* = 1 . . . *n* – samples

- mutual information
- Pearson correlation

C. Allefeld, M. Müller, J. Kurths. Eigenvalue decomposition as a generalized synchronization cluster analysis. International Journal of Bifurcation and Chaos, 17(10):3493–3497, 2007.

Example results

- Asymmetric auditory response (MEG)
- (delayed) correlation matrix; eigenspectrum, n-n spacing distribution, etc. (GOE)

$$C_{\alpha,\beta}(\tau)=\frac{\sum_i [x_\alpha(t_i)-\overline{x}_\alpha][x_\beta(t_i+\tau)-\overline{x}_\beta]}{\sqrt{\sum_i [x_\alpha(t_i)-\overline{x}_\alpha]^2 \sum_j [x_\beta(t_j+\tau)-\overline{x}_\beta]^2}},$$

J. Kwapień, S. Drożdż, A. A. Ioannides, Phys Rev E 62 (2000) 5557-5564
S. Drożdż, J. Kwapień, A. A. Ioannides, Acta Phys Pol B 42 (2011) 987-999

Example results

- Several papers on detection of epileptic seizure (EEG, ECoG)
- Mostly equal-time correlation matrix: eigenspectrum, n-n spacing distribution, number variance

P. Šeba. Random matrix analysis of human eeg data. Phys. Rev. Lett., 91 (2003) 198104,

H. Osterhage, S. Bialonski, M. Staniek, K. Schindler, T. Wagner, C. E. Elger, K. Lehnertz. Bivariate and multivariate time series analysis techniques and their potential impact for seizure prediction. In: *Seizure prediction in epilepsy: from basic mechanisms to clinical applications*, (2008) 189–208.

Our data

Resting state

- Assumed to be **spontaneous** or intrinsic brain activity
- Rest coactivates the same regions as task
- The patterns align with known anatomical and functional pathways

Correlated fluctuations at rest in many functional systems :

- visual and auditory, memory, language, attention, and control systems

Data acquisition

The data were recorded with:

- 65 channels (64 plus EOG)
- active electrodes (actiCHamp, Brain Products)
- 2500 Hz sampling rate
- reference electrode: Cz

We analyse EEG recordings of healthy humans in three states:

- REO: 3 minutes of resting state with eyes open,
- REC: 3 minutes of resting state with eyes closed,
- Flanker: 10 minutes (384 stimuli)

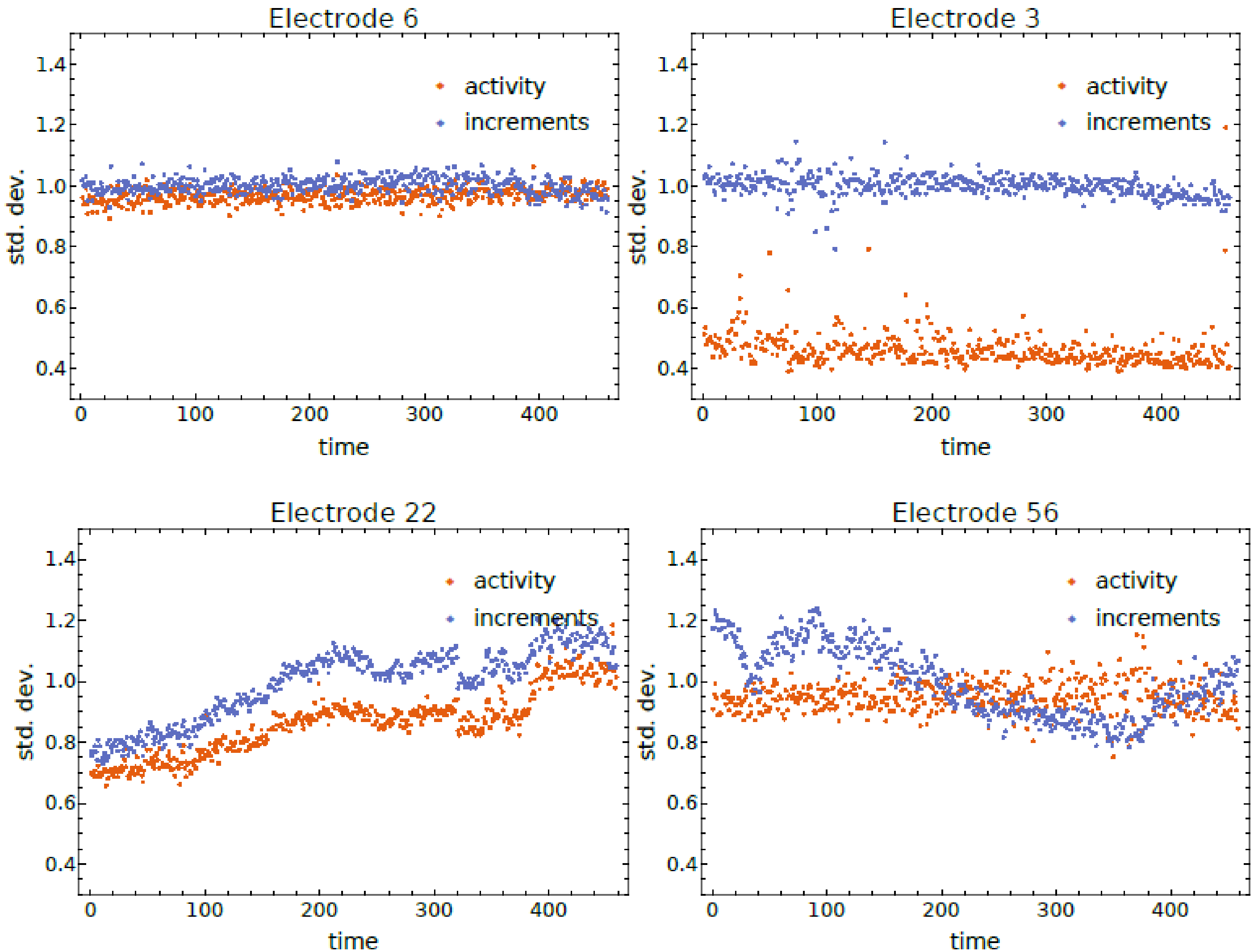
Data preprocessing

- bandpass filter: 1-1245 Hz, bandstop: 50 Hz
- NO: downsampling, re-referencing, removing poor quality channels, poor quality data intervals, or other artefacts.

How these effects are manifested in eigenspectra and eigenvectors of correlation matrices.

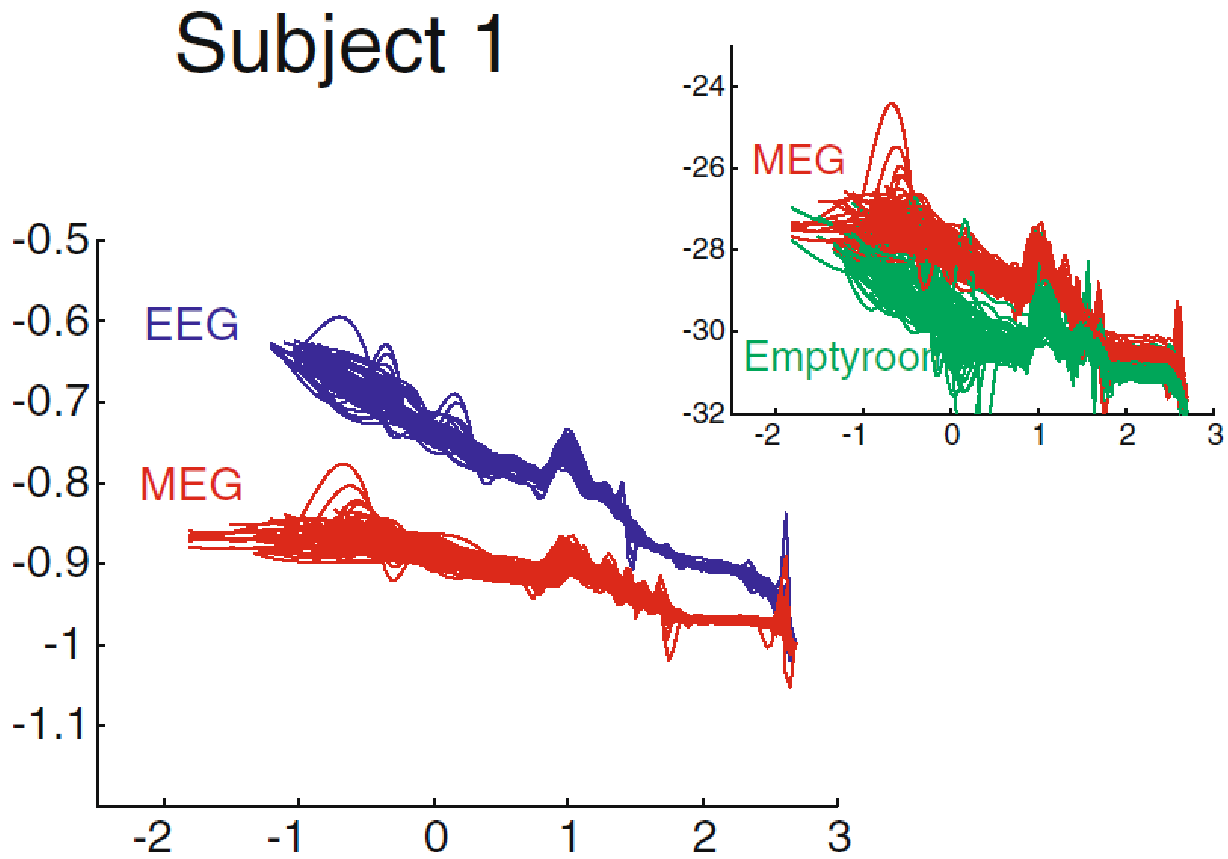
Properties of EEG signal

- Non-stationarity



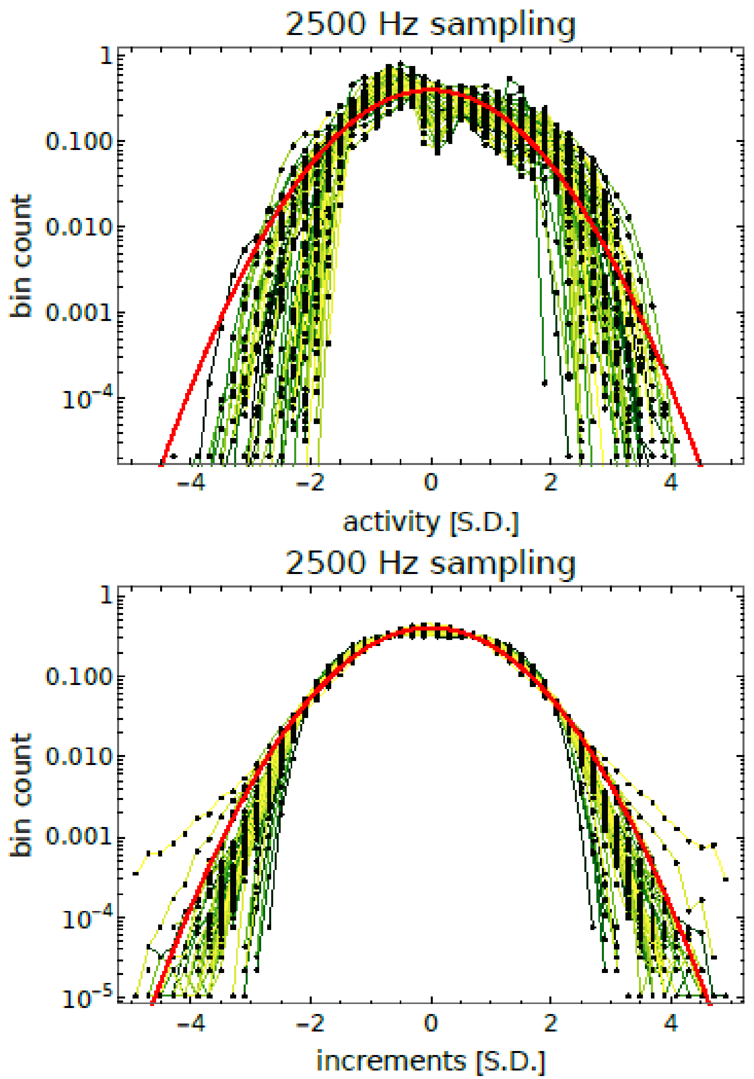
Properties of EEG signal

- Non-stationarity ($1/f^{a}$, $1<a<2$)



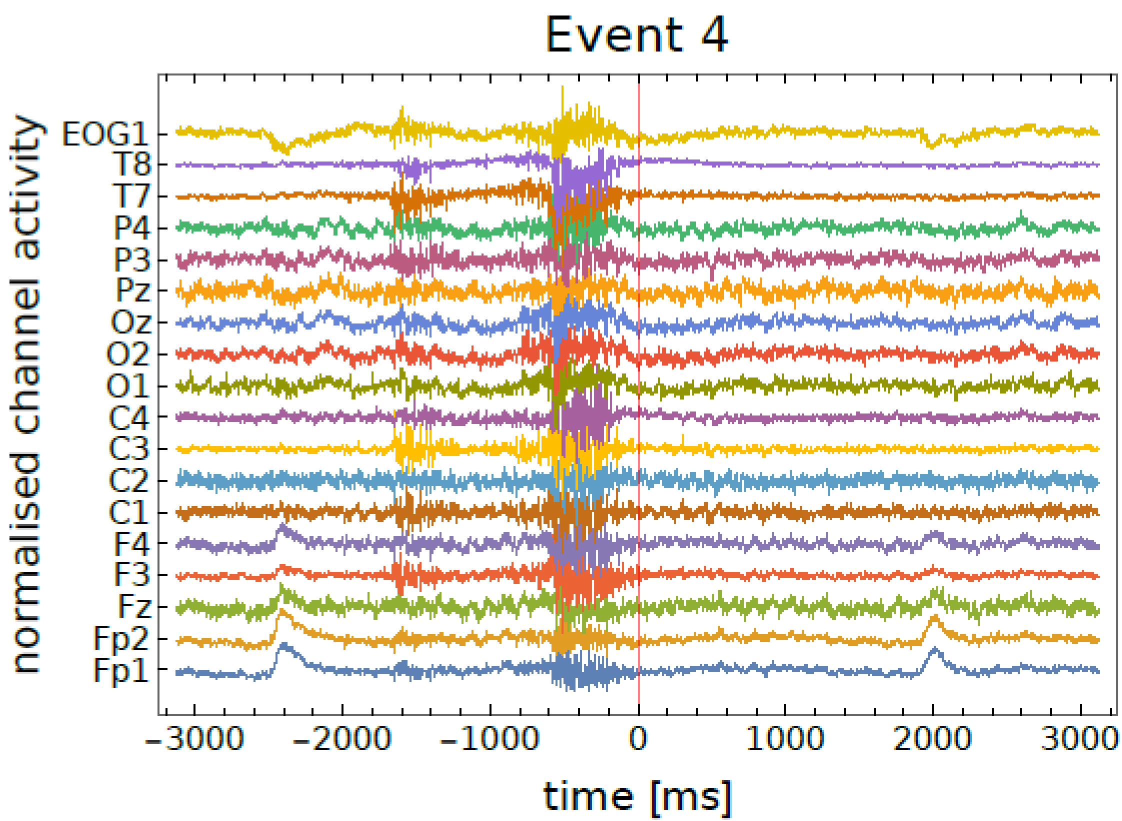
Properties of EEG signal

- Non-stationarity ($1/f^{\alpha}$)
- Non-gaussianity



Properties of EEG signal

- Non-stationarity ($1/f^{\alpha}$)
- Non-Gaussianity
- Artifacts



Results

Cross-correlations

- Equal-time Pearson correlation estimator

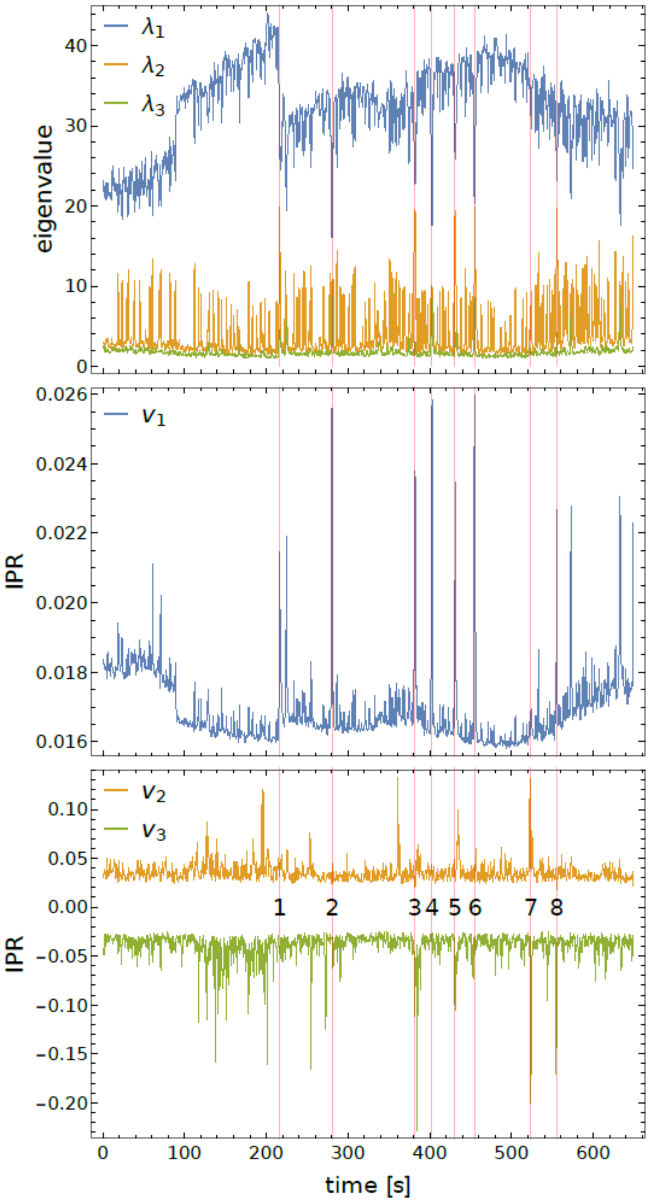
$$C_{ij} = \frac{1}{T}\sum_{t=1}^{T} x_{i,t}\bar{x}_{j,t},$$

i,j=1,…, *N* – channels, *t*=1,…, *T* – time, x_i – zero mean, unit variance

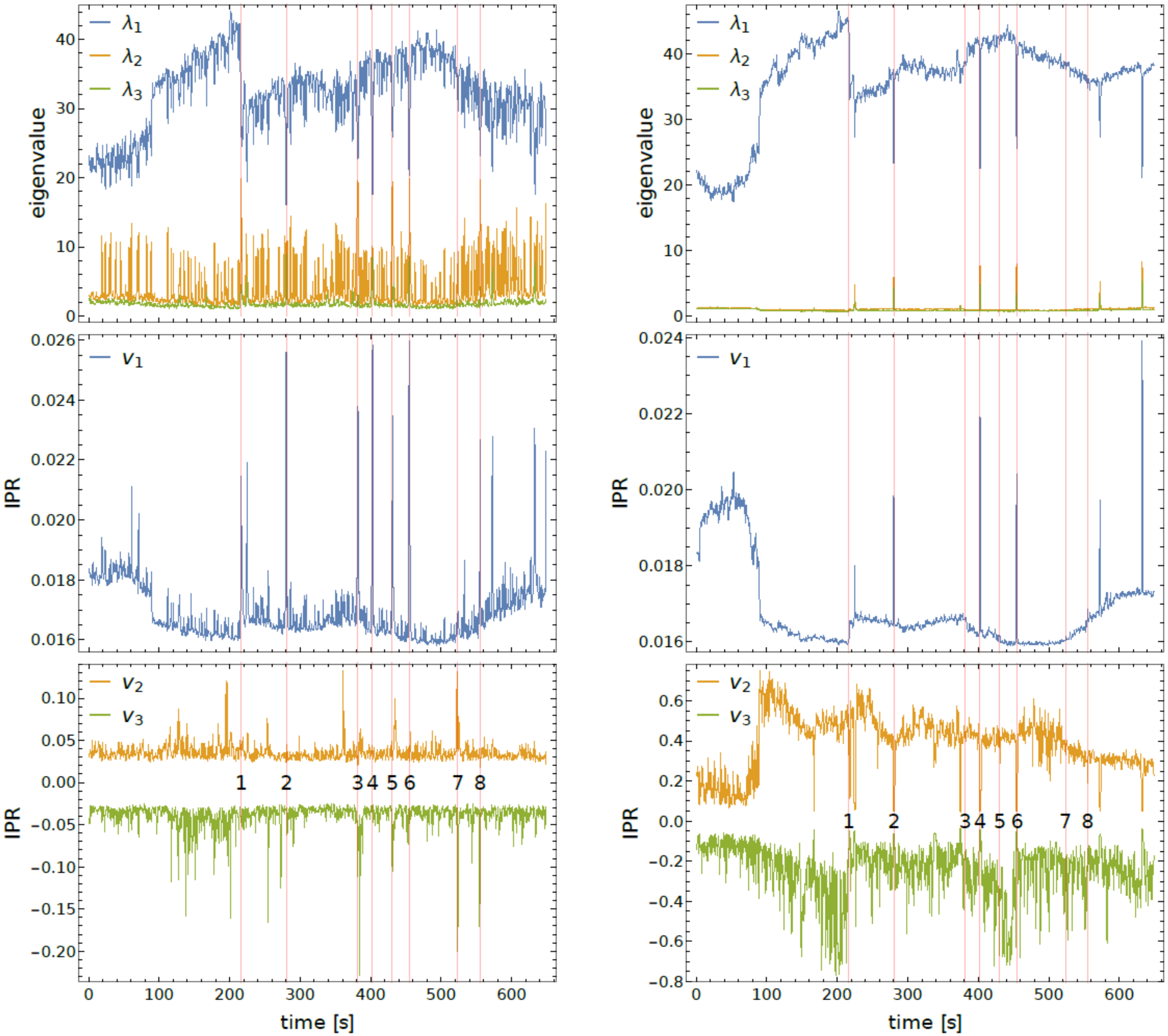
- Lagged correlations

$$C_{ij}^{\tau} = \frac{1}{T-\tau}\sum_{t=1}^{T-\tau} x_{i,t}x_{j,\bar{t}+\tau}$$

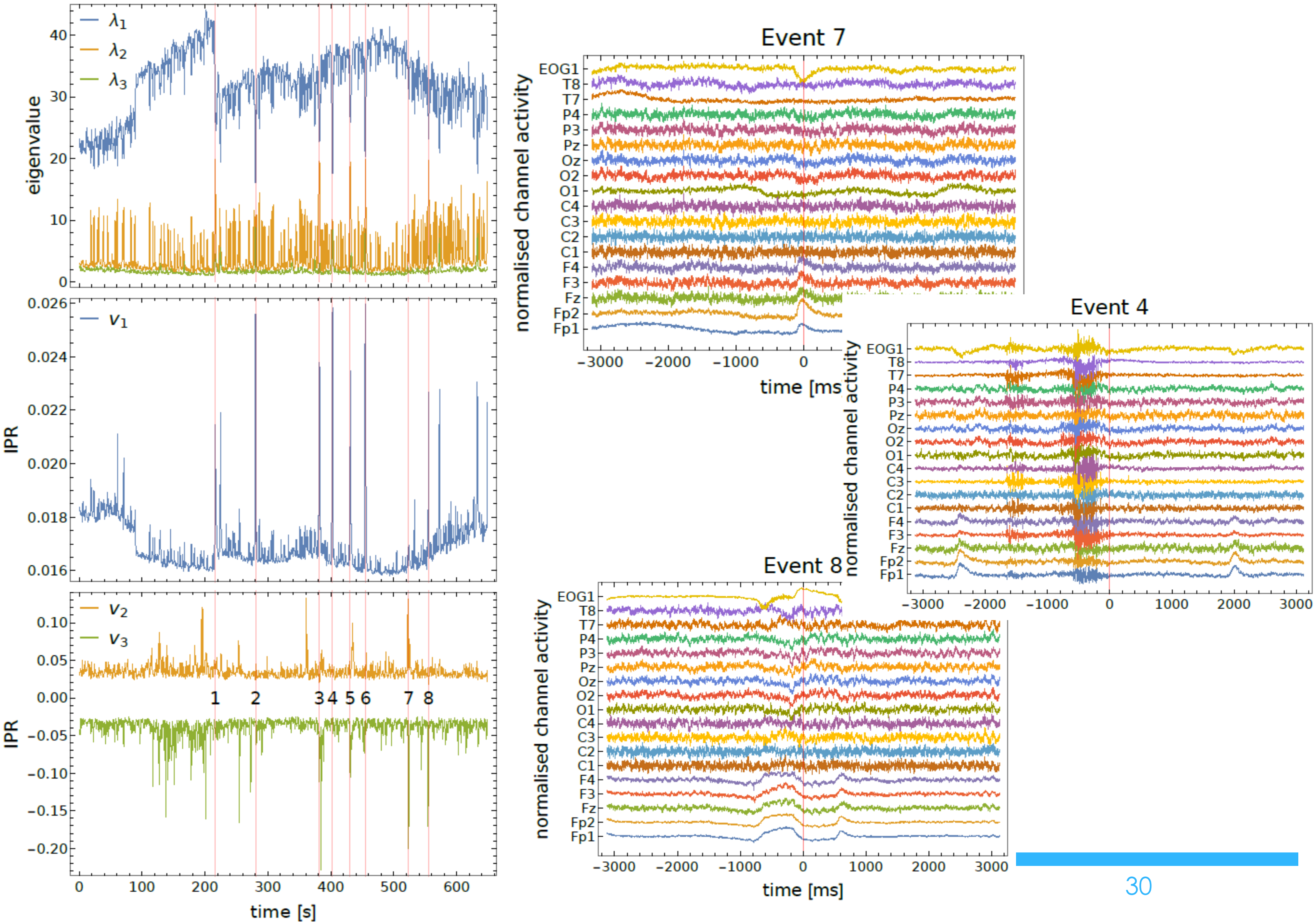
Largest eigenvalues



Largest eigenvalues



Largest eigenvalues



Equal-time spectra

- Equal-time Pearson correlation estimator

$$C_{ij} = \frac{1}{T}\sum_{t=1}^{T} x_{i,t}\bar{x}_{j,t},$$

For iid Gaussian random variables, large N, T limit – well known.

- Spectral density given by Marčenko-Pastur distribution

$$\rho_{\mathbf{c}}(x) = \frac{1}{2\pi r x}\sqrt{(\lambda_+ - x)(x - \lambda_-)}$$

with $\lambda_\pm = (1 \pm \sqrt{r})^2$ and $r = N/T$

Equal-time spectra

Spectral densities are also known for:

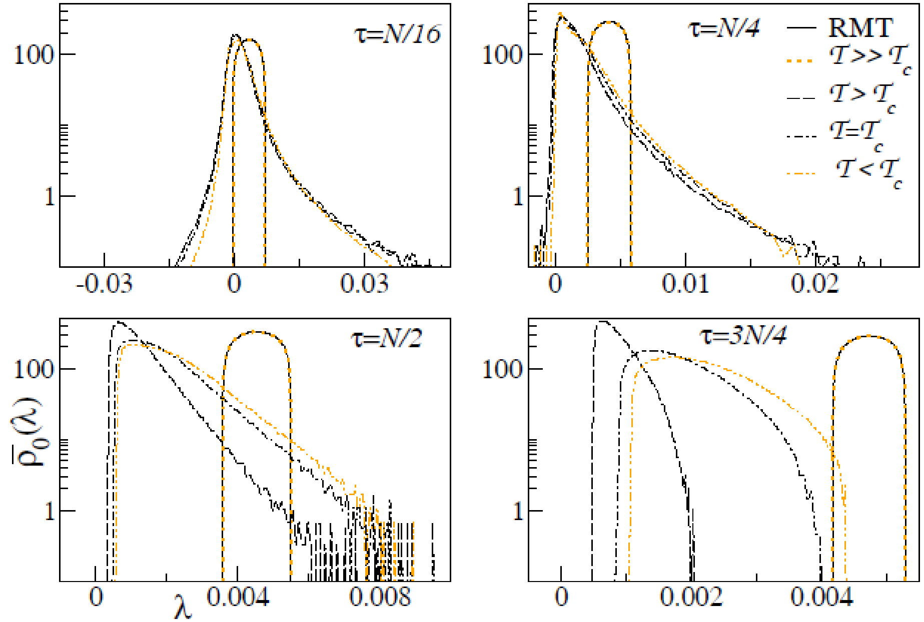
- ARMA processes

$$x_{i,t} - \sum_{\alpha=1}^{q_1} a_\alpha x_{i,t-\alpha} = \sum_{\beta=0}^{q_2} b_\beta \epsilon_{i,t-\beta}$$

Z Burda, A Jarosz, M A Nowak, M Snarska, *A random matrix approach to VARMA processes*. New J. Physics 12 (2010) 075036

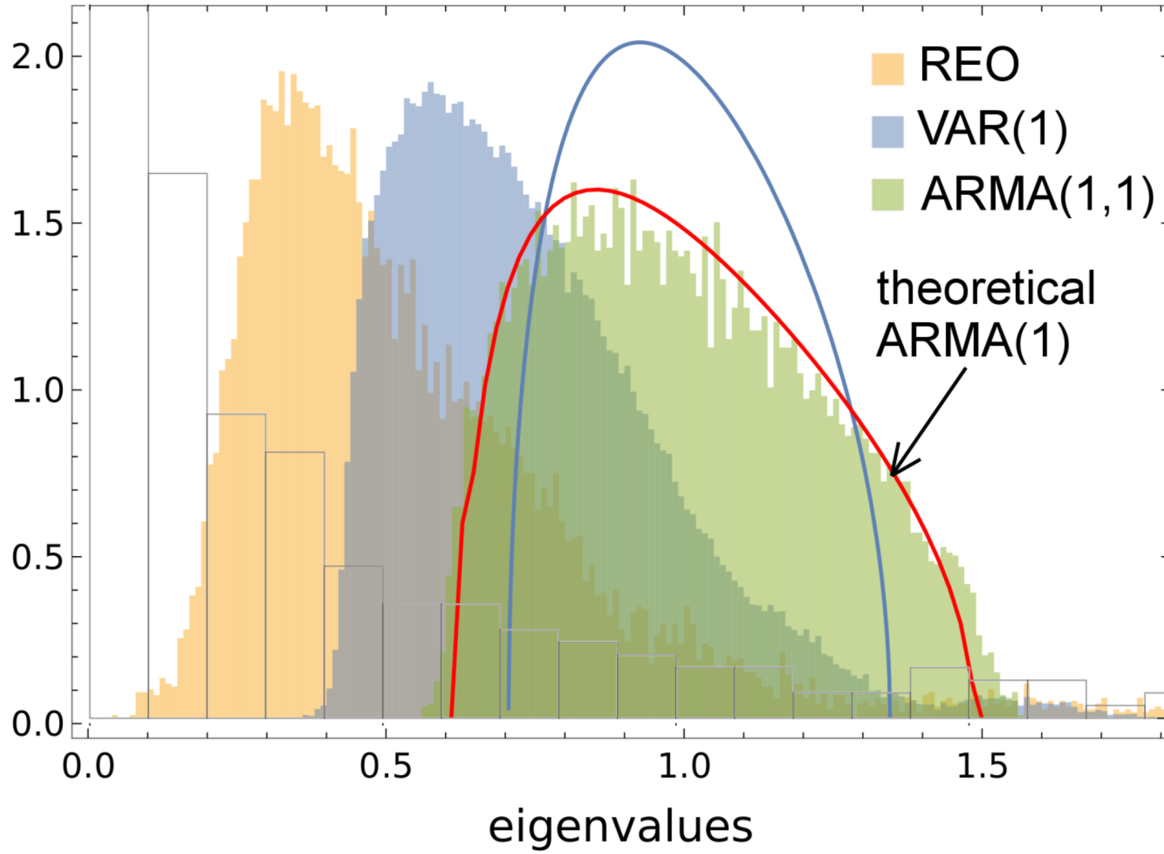
- Ising model

TP Vinayak, B Buca, TH Seligman, *Spectral analysis of finite-time correlation matrices near equilibrium phase transitions*, EPL 108 (2014) 20006



Equal-time spectra

- Sampled with 1s data segments
- Highly skewed due to „market mode"

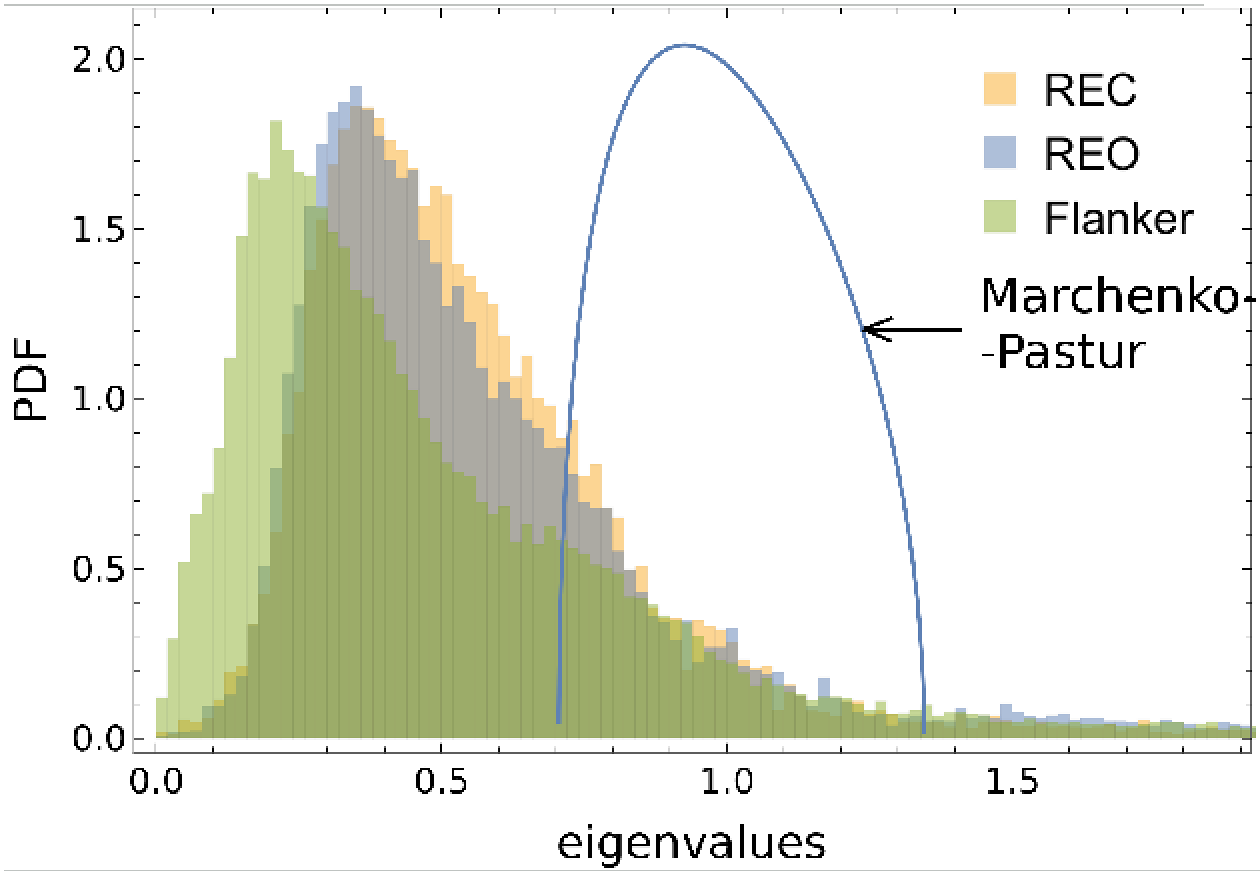


- At least VAR(q) needed to get collective modes

$$x_{i,t} - \sum_{\alpha=1}^{q} \sum_{j=1}^{N} A_{\alpha,ij} x_{j,t-\alpha} = b_0 \epsilon_{i,t}$$

Equal-time spectra

- Difference between task and resting state



Lagged correlations

- Lagged correlations

$$C_{ij}^{\tau} = \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} x_{i,t} x_{j,t+\tau}$$

Shown to be contained in a circle of radius s_{ext}

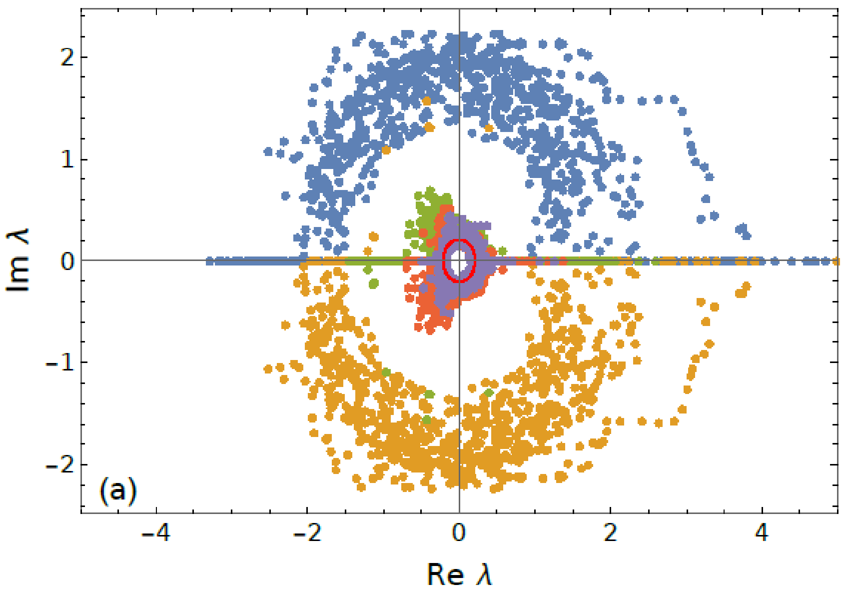
$$\sum_{k=1}^{M-1} \left(\frac{\alpha r}{s_{\mathrm{ext}}} \right)^{2k} (1 - k\beta) = r,$$

where $r = N/T, \beta = \tau/T, \alpha = (1-\beta)^{-1}, M = \lceil T/\tau \rceil$

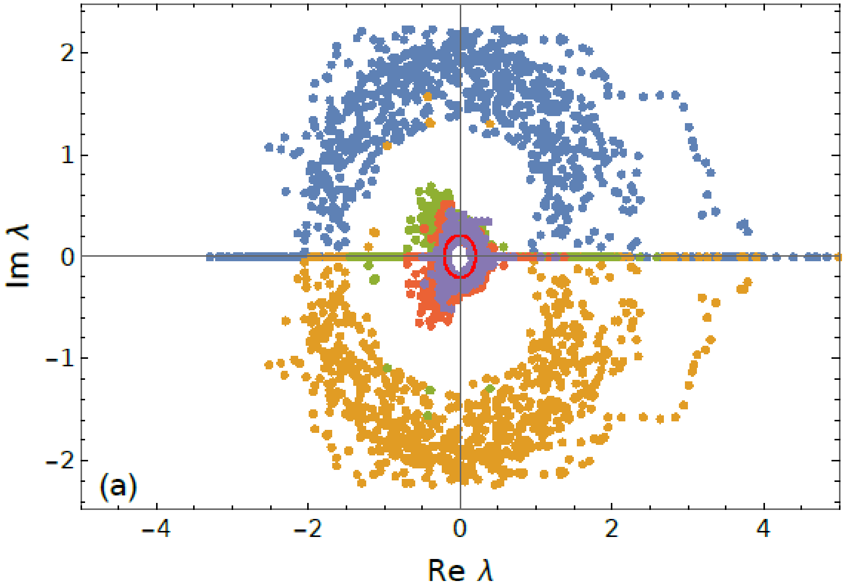
W Tarnowski, *Free Random Variable Approach to lagged correlation matrices,* MSci Thesis, Jagiellonian University (2016)

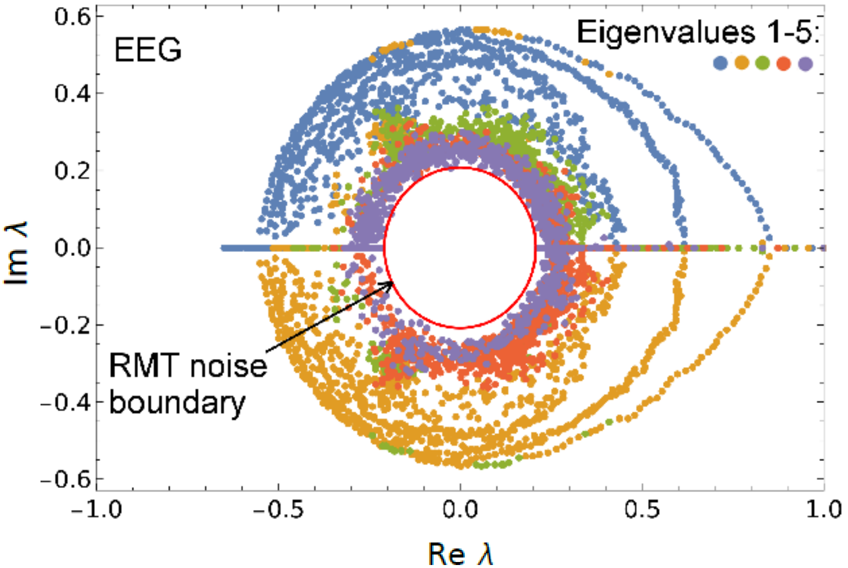
M A Nowak, W Tarnowski, *Spectra of large time-lagged correlation matrices from random matrix theory*, J. Stat. Mech. (2017) 063405. ArXiv: 1612.06552

Lagged spectra



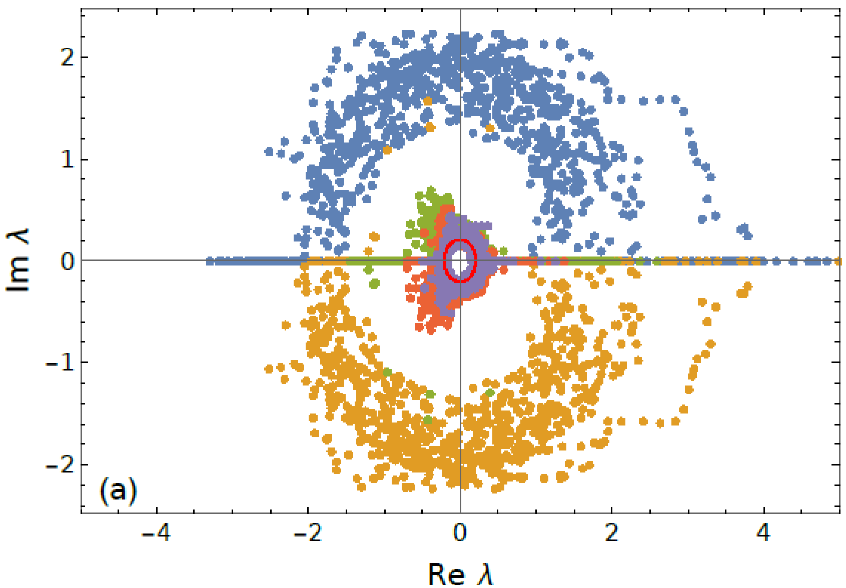
Lagged spectra

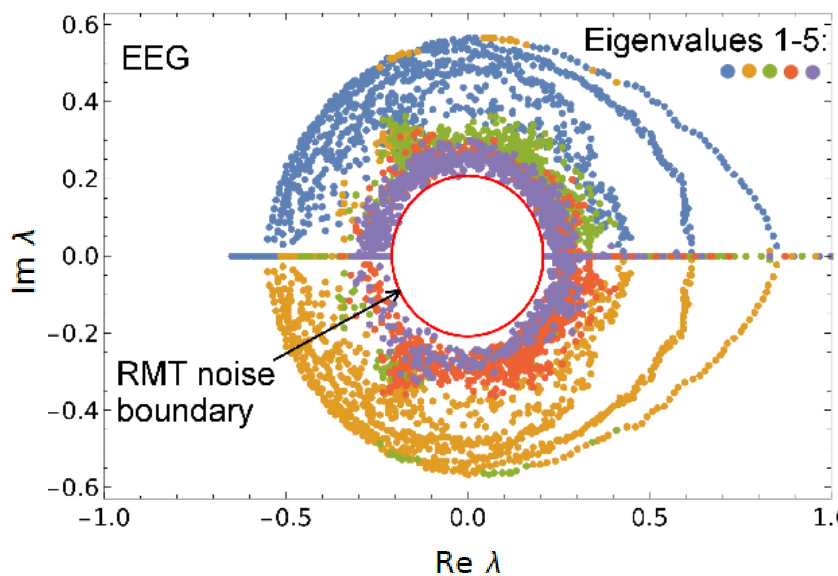




- After whitening

Lagged spectra



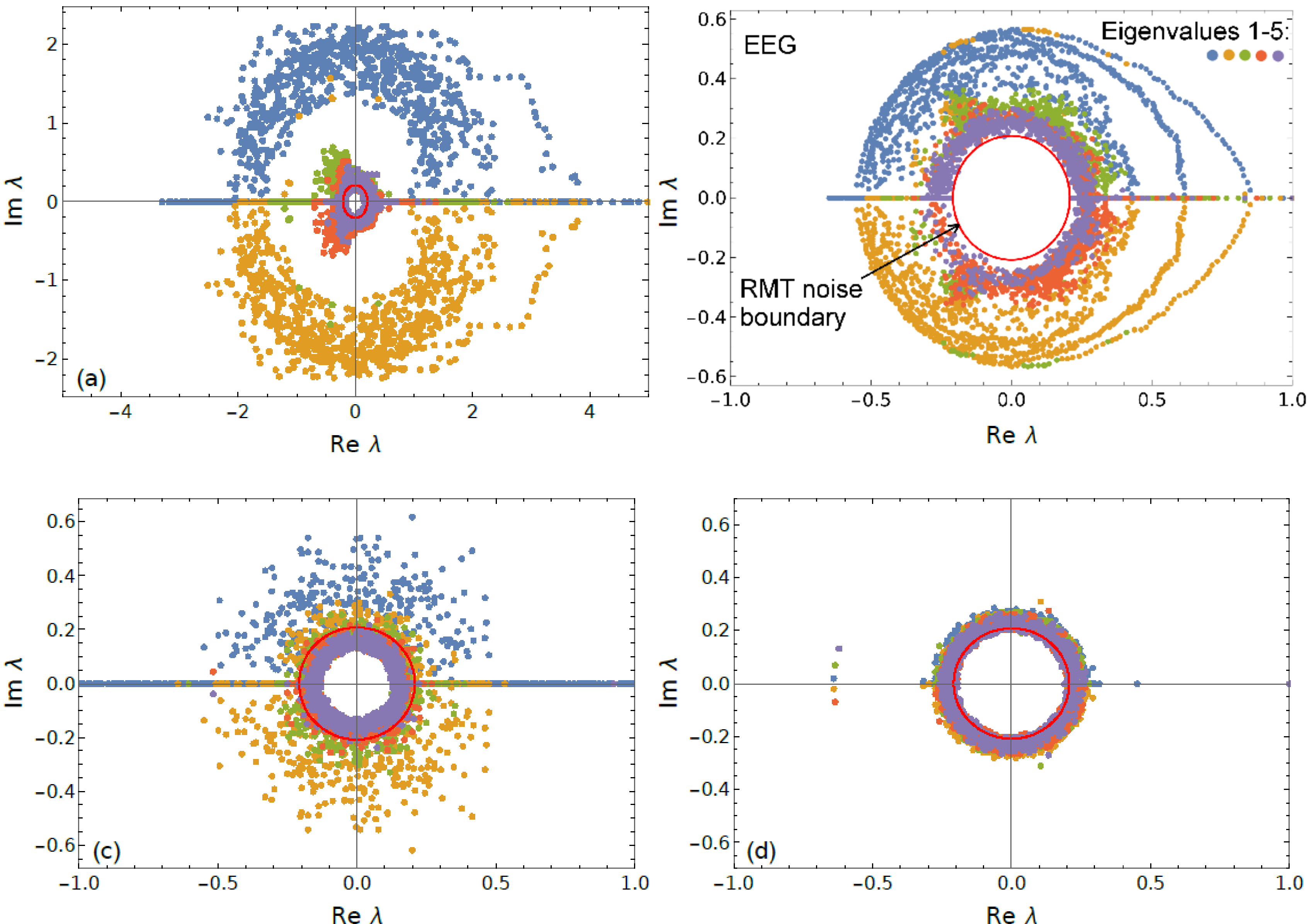


- shown lag = 0-400 ms
- two semi-circle takes 100 ms
- common frequency oscillatory signals produce

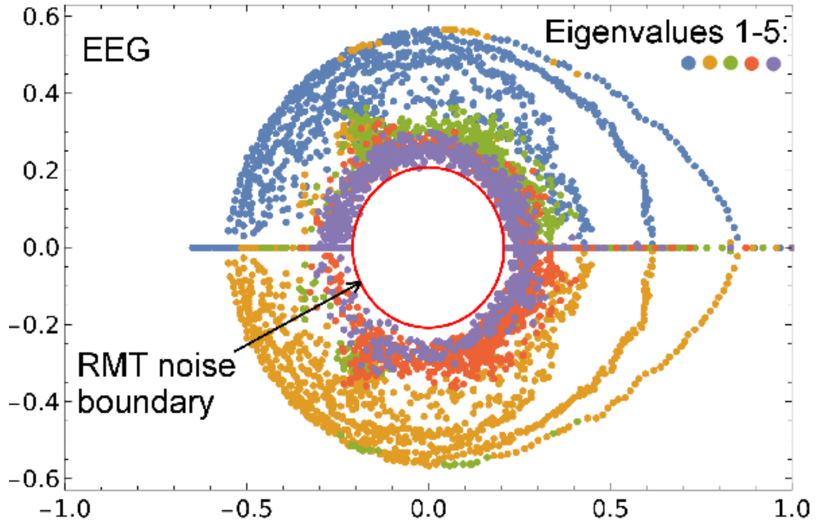
$$C^{\tau}_{ij} \sim \frac{1}{2}\cos(\phi_i - \phi_j - 2\pi\tau\omega),\ \lambda_1, \lambda_2 \sim a\cos(2\pi\tau\omega) + ib\sin(2\pi\tau\omega)$$

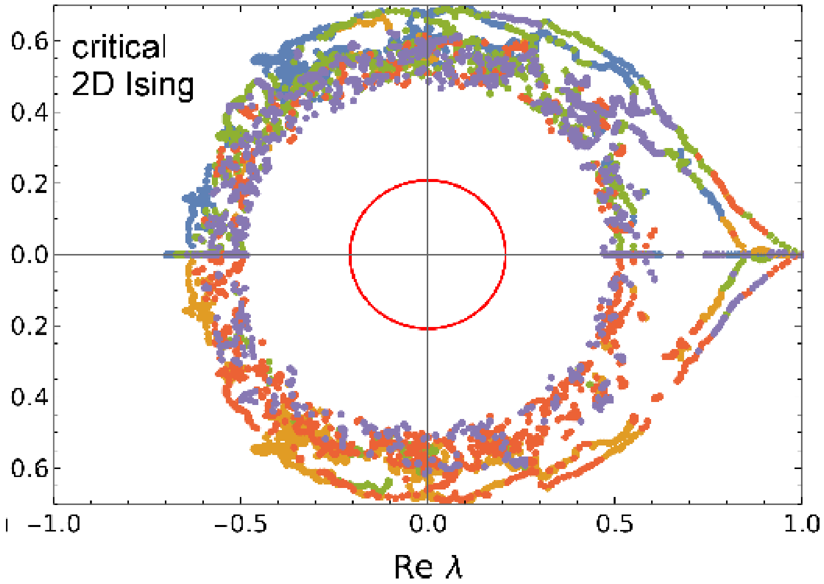
  - rediscovered alpha rhythm (8-12 Hz)

Lagged spectra



Lagged spectra





Conclusions

- Strong collective behaviours visible both in the largest eigenvalues and **eigenvectors**
- detecting and classifying events (?)
- cognitive state **»** change in the whole eigenspectrum
- whitening the series brings eigenvalues out of the noise region
- Open question: determine the right null model (critical, temporally auto-correlated).

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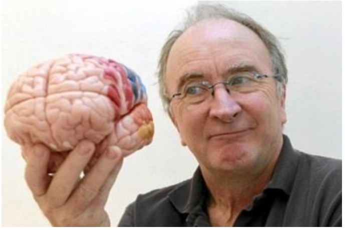
MA Nowak



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Grant: 2015/17/D/ST2/03492

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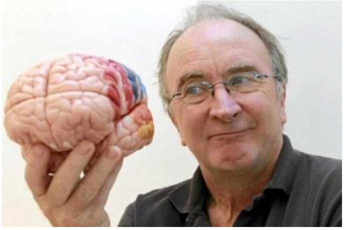

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Thank you!

Questions?

NATIONAL SCIENCE CENTRE
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Grants: 2015/17/D/ST2/03492, 2011/02/A/ST1/00119

Thank you!

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